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On the Theory of Spin-Two Particles.

ADEL DA SILVEIRA
Colégio Pedro II - Rio de Janeiro, D.F., Brazil

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Summary. — The gauge transformation of second kind, for the equation of spin-two particles proposed previously, is obtained. It is found that the Einstein equation coincides with the spin-two equation, in the case of weak gravitational field.

1. – Introduction.

It has been pointed out recently [1] that it is possible to get an equation for spin-two particles, from which the subsidiary conditions that limit the number of independent wave functions can be obtained. As in the Pauli-Fierz theory of such particles [2], we should expect the invariance of the equation with respect to a gauge transformation in the case of zero-mass, as well as its reduction to the Einstein equation of the general relativity for the case of weak gravitational field. This reduction is indeed obtained by means of a convenient additional condition, which results from the suitable choice of the gauge.

(*) Liberato dall'Autore il 18 Febbraio 1956.

[1] A. DA SILVEIRA: *Phys. Rev.*, **97**, 1144 (1955).

[2] W. PAULI and M. FIERZ: *Proc. Roy. Soc.* (London) A **173**, 211 (1939).

2. – Equation and Subsidiary Conditions.

Let us start with the Lagrangian L for spin-two particles in interaction with the electromagnetic field:

$$(1)\qquad L = -\tfrac{1}{2}m^2 A^*_{ik}A_{ik} + \tfrac{1}{6}m^2 A^*_{kk}A_{jj} - \tfrac{1}{2}\,\partial^*_j A^*_{ik}\,\partial_j A_{ik} + \tfrac{1}{6}\,\partial^*_i A^*_{jj}\,\partial_i A_{rr} + \\ + \partial^*_r A^*_{rk}\,\partial_j A_{jk} - \tfrac{1}{6}\,\partial^*_i A^*_{ik}\,\partial_k A_{jj} - \tfrac{1}{6}\,\partial^*_k A^*_{jj}\,\partial_i A_{ik}\,.$$

In the Lagrangian (1), A_{ik}, the wave function is supposed to be symmetric and A_{jj} is the spur of A_{ik} (i and k run from one to four). A^*_{ik} is the complex conjugate of A_{ik}, for i, k=1, 1,2 3. We define $-A^*_{4k}$ as the complex conjugate of A_{4k}, in order to obtain the same equation for A_{ik} and A^*_{ik}. (x_i) stands for (x, y, z, it). We are using the system such that $\hbar=1$, $c=1$. φ_i is the electromagnetic potential and

$$(2)\qquad \partial_i = \frac{\partial}{\partial x^i} - ie\varphi_i\,,$$

$$\partial^*_i = \frac{\partial}{\partial x^i} + ie\varphi_i\,.$$

As in the Pauli-Fierz paper, we shall call additional conditions those which are not obtained from the principal equation. The others, as (4) and (5), are called subsidiary conditions.

From the function L given in (1), on account of the simmetry of the A_{ik} we get the equation:

$$(3)\qquad (\Box - m^2)A_{ik} - \partial_i\partial_r A_{rk} - \partial_k\partial_r A_{ri} + \tfrac{1}{6}(\partial_i\partial_k + \partial_k\partial_i)A_{jj} + \\ + \tfrac{1}{3}\delta_{ik}\,\partial_r\partial_j A_{jr} - \tfrac{1}{3}\delta_{ik}\Box A_{jj} + \tfrac{1}{3}\delta_{ik} m^2 A_{jj} = 0\,,$$

where δ_{ik} is the Kronecker delta and $\Box$ is the Dalembertian $\partial_i\partial_i$. Contracting i and k in (3) and by applying ∂_i to the same equation, we obtain

$$(4)\qquad m^2 A_{jj} = 2\,\partial_i\partial_k A_{ik}\,,$$

and

$$(5)\qquad m^2\,\partial_i A_{ik} = B_k\,,$$

where B_k depends upon the electromagnetic φ_i. By means of (4), the equation (3) can be put in the form given in the reference [1].

The conditions (4) and (5) as well as the symmetry of the A_{ik} are the eleven relations necessary to reduce the sixteen A_{ik} to the five independent wave

functions. If $\varphi_i \equiv 0$, the equations (3), (4) and (5) reduce to those proposed by PAULI and FIERZ for the free particle. One must observe, however, that if we make the electromagnetic φ_i go to zero, the first member of (5) is not zero in view of the potential derivatives. The free particle equations are not obtained but the number of independent functions remains the same.

3. – Particle with $m = 0$.

If the mass of the particle is zero, however, we cannot reduce (3), (4) and (5) to the vacuum values, the theory beeing slightly different. Let us start wth the equation:

$$\Box A_{ik} - \partial_i \partial_r A_{rk} - \partial_k \partial_r A_{ri} + \tfrac{1}{3}\, \partial_i \partial_k A_{jj} - \tfrac{1}{3}\, \delta_{ik} \Box A_{jj} = 0, \tag{3'}$$

for particles in the vacuum (from now on ∂_i means only partial derivative and not the definition given in (2)), easily obtained from (3). Besides a relation analogous to (4) with $m = 0$, which is got from (3′), we introduce the additional relation

$$\partial_k A_{jj} = 6 \partial_i A_{ik}\,, \tag{6}$$

which is a consequence of a particular choice of the gauge, as we shall see later. It is easy to see by means of (6) that the last term of (3′) is zero:

$$\Box A_{jj} = 0\,. \tag{7}$$

Now, if we perform the transformation

$$\gamma_{ik} = A_{ik} - \tfrac{1}{3}\, \delta_{ik} A_{jj}\,,$$

(3′), with the help of (7) reduces to

$$\Box \gamma_{ik} - \partial_i \partial_r \gamma_{rk} - \partial_k \partial_r \gamma_{ri} + \partial_i \partial_k \gamma_{jj} = 0\,, \tag{8}$$

which is the Einstein equation for the weak gravitational field in vacuum [3] (i.e. $g_{ik} = \delta_{ik} + \gamma_{ik}$). The new transformation

$$\gamma'_{ik} = \gamma_{ik} - \tfrac{1}{2} \delta_{ik} \gamma_{jj}\,, \tag{9}$$

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[3] A. EINSTEIN: *The Meaning of Relativity*, third edition, (Princeton, 1950), p. 86.

as well as the condition

$$\partial_i \gamma'_{ik} = 0\,, \tag{10}$$

are imposed to reduce the first member of (8) to its first term. The relations (9) and (10) are consistent with (6).

The equation (3′) is invariant with respect to the gauge transformation of second kind:

$$A'_{ik} = A_{ik} + A^0_{ik}\,, \tag{11}$$

where

$$A^0_{ik} = \partial_k f_i + \partial_i f_k - 2\delta_{ik}\partial_j f_j\,, \tag{12}$$

the f_i being arbitrary functions. With the help of this gauge, γ_{ik} is transformed into

$$\gamma_{ik} + \partial_k f_i + \partial_k f_k\,,$$

and γ'_{ik} defined by (9) goes on

$$\gamma'_{ki} + \partial_k f_i + \partial_k f_k - \delta_{ik}\partial_j f_j\,.$$

If we take into account (11) and (12) as well as the definition of γ_{ik}, we can see that (6) and (10) are satisfied by a suitable choice of the f_i:

RIASSUNTO (*)

Si ottiene la trasformazione di « gauge » di seconda specie per l'equazione delle particelle con spin 2, precedentemente proposta. Si trova che l'equazione di Einstein coincide con l'equazione dello spin 2, nel caso di un campo gravitazionale debole.

(*) *Traduzione a cura della Redazione.*

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Relativistic Invariance in Quantum Mechanics (*).

E. P. Wigner

Palmer Physical Laboratory - Princeton, N. J.

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Summary. (*). — A detailed analysis is made of the theoretical possibilities of constructing a quantum mechanics and consequently of a description of the elementary particles, based on a definition of the postulates of relativistic invariance and then on the simmetry properties connected with the complete Lorentz group, including displacements in space and time and inversions of both the space and the time coordinates. It is then pointed out how the validity of every conclusion deriving from simmetry considerations depends essentially from the fundamental problem of the « measurability » of the field quantities.

(*) *Editor's care.*

1. – Introduction.

Once quantum mechanics has proved to be able to give, in principle, a satisfactory account of everyday phenomena [1], its reconciliation with, and adjustment to, the principle of relativity became one of the foremost objectives of theoretical physics. The period began around 1925; the articles of Dirac on the relativistic equation of the electron [2] and of Heisenberg and Pauli on quantum electrodynamics [3] were the first two milestones on the road toward it. The full aim of the reconciliation of the two most important physical theories of our century has not yet come to a conclusion although, more recently, the problem of elementary particles begins to compete for the first

(*) Paper presented at the International Conference on Elementary Particles held at Pisa, June 1955.

([1]) P. A. M. Dirac: *Proc. Roy. Soc.*, **123**, 714 (1929).

([2]) P. A. M. Dirac: *Proc. Roy. Soc.*, **117**, 610 (1928); **118**, 351 (1928).

([3]) W. Heisenberg and W. Pauli: *Zeits. f. Phys.*, **56**, 1 and **59**, 168 (1929); P. Jordan and O. Klein: *Zeits. f. Phys.*, **45**, 751 (1927).

place in the interest of the theoretical physicist. In the period since 1925, a wealth of ideas has been formed and it would be vain for me to attempt a full review of them. I must ask forgiveness of those whose thinking I appreciated too little or failed to understand. All of us share the fault of appreciating ideas of others less than those of ourselves.

Because of the wealth of problems, points of view and results on the role of symmetry in quantum mechanics, I shall have to limit myself rigidly in the choice of the subjects to be discussed and even as far as this subject is concerned, I shall discuss principally only one point of view. The point of view which I shall adopt is that the problems of physics are still rather far from their solution and that the role of symmetry and invariance is that of a guide in the development of the proper physical concepts, rather than something that one simply reads off from the ready equations. To illustrate this, I shall not say that physical equations are invariant under rotations in space because the Hamiltonian contains only distances and the absolute values of the momenta. Rather, my point of view would be to search for those Hamiltonians which give a physical theory that is invariant under spatial rotations and other relativistic transformations. This point of view will be the one adopted throughout and my endeavour will be to be quite consistent in this.

Second, I shall restrict myself entirely to the symmetry demanded by the special theory of relativity; that is, the group which will underly my considerations will be the Lorentz group, including displacements in space and time and inversions of both the space and the time coordinates. It was after some hesitation that I decided to restrict myself in this way because the attempt to incorporate at least some of the ideas of general relativity into quantum mechanics is both a challenging and a fascinating one.

Third, I shall restrict myself to the consideration of the symmetry elements of space-time. This will exclude, in particular, the symmetry properties of interaction. This applies even to the symmetry between positive and negative charges—so very important a symmetry and so natural to consider for every physicist (4). I did this because it is hardly possible to know in this regard where to stop: from charge conjugation one is led to isotopic spin and to the generalizations of these concepts to explain properties of the semistable particles found in cosmic ray phenomena (5). It was reassuring, therefore, to read in our program that the theories of the semistable particles will be treated by other speakers.

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(4) W. Furry: *Phys. Rev.*, **51**, 125 (1937); L. Wolfenstein and D. G. Ravenhall: *Phys. Rev.*, **88**, 279 (1952); J. Schwinger: *Phys. Rev.*, **82**, 914 (1951); G. Lüders: *Zeits. f. Phys.*, **133**, 325 (1952); *Kon. Danske Vidensk. Selskab, Math.-fys. Medd.*, **28**, No. 5 (1954).

(5) W. Heitler: *Proc. Roy. Irish Acad.*, **51**, 33 (1946).

Finally, even as far as the subject of space-time symmetry is concerned, I will try to present a point of view rather than detailed results.

2. – Relativistic Invariance.

Let me begin with the question of what we mean by relativistic invariance. In macroscopic, that is non-quantum, theory this concept is easily defined. To do it, the concept of the complete description of a physical system is useful. This shall consist of a full specification of the paths of all particles, together with a full description of all fields at all points of space-time. Given such a complete description of a physical system in a coordinate system, the equations of motion permit one to determine whether the complete description is compatible with them: the equations of motion give a criterion whether the physical system could have behaved in the way specified by the complete description. The principle of relativistic invariance, or of any invariance, then makes three postulates. These have been stated, with admirable clarity, a short time ago by R. HAAG [6] in an article which I have not yet seen in print. They are

(*a*) It should be possible to translate a complete description of a physical system from one coordinate system into every equivalent coordinate system.

(*b*) That the translation of a dynamically possible description be again dynamically possible. Expressed in a somewhat more simple language: a succession of events which appears possible to one observer should appear possible also to any other observer.

(*c*) That the criteria for the dynamical possibility of complete descriptions be identical for equivalent observers.

It is not customary to formulate the principle of invariance as explicitly as I just did. In particular, the possibility of translating the description of the behavior of a physical system from one coordinate system into another is usually taken for granted. The same applies to postulate (*b*). The remaining postulate can then be stated as the invariance of the equations of motion. However, if one wishes a precise formulation of the principle of invariance in quantum theory, the translation of the languages of the different observers becomes a crucial point. It may be well, therefore, to emphasize the problem of translation already in macroscopic, that is, classical theory.

The problem of translation between different coordinate systems, or be-

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[6] R. HAAG: mimeographed notes (1953), unpublished.

tween different observers, becomes so much more serious in quantum theory because the basic concept of quantum theory is that of observation by individual observers (7). If the observation of one observer can not be translated into the language of other observers, the principle of relativity becomes practically meaningless. This would be true, for instance, if the observation were an instantaneous process but extending over a finite part of space (or over all space), as would correspond to most measurements considered in non-relativistic quantum theory. Since the t = constant planes of space-time are different for different observers, there is no obvious way to translate the observations of observers in relative motion with respect to each other. In order to make a translation possible, one usually introduces a wave function which summarizes the results of all previous measurements. One then can formulate the translation from one coordinate system into another in terms of the wave function or state vector.

The wave function or state vector also contains all possible information on the probabilities of the outcomes of future experiments. The translation therefore relates the expectations of different observers regarding the outcomes of experiments which they may carry out on a system. One sees that the translation has much more content in quantum theory than in classical theory; it answers a much more intricate question. One also sees that the fewer experiments are permissible « in principle », the easier it will be to satisfy the principle of invariance, the less meaningful such a principle will become.

All wave functions or state vectors which are constant multiples of each other are often said to form a ray. If all self adjoint operators are observable, it is possible to distinguish between any two rays in Hilbert space, that is between any two vectors which are not constant multiples of each other. If not all self adjoint operators are observable, as is undoubtedly the case (8), it may not be possible to distinguish every pair of rays in Hilbert space. This will make it easier to satisfy the principle of relativistic invariance but will make it also a bit less significant. I wish therefore to devote a few minutes to the discussion of observables, or to the equivalent question of the possibility of distinguishing between state vectors even though this discussion will prove somewhat futile.

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(7) W. HEISENBERG: *Zeits. f. Phys.*, **43**, 172 (1927) and *Physikalische Prinzipien der Quantentheorie* (Leipzig, 1930); N. BOHR: *Nature*, **121**, 580 (1928); J. v. NEUMANN: *Mathematical Foundations of Quantum Mechanics* (Princeton, 1955), Chapter VI.

(8) Cf. G. WICK, A. S. WIGHTMAN and E. P. WIGNER: *Phys. Rev.*, **88**, 101 (1952).

3. – Heisenberg and Schrödinger Representations.

There are two ways in which quantum mechanics can be formulated: the Heisenberg picture and the Schrödinger picture (⁹). The principal concepts of the Heisenberg picture are certain observables and measurements. These are the ones which change in time and are subject to the laws of motion. The state vector, which lives a relatively withdrawn life, remains unchanged. In the Schrödinger picture, on the contrary, the wave function is the principal concept, it is the one which changes in time while the operator of a measurement remains the same, no matter at what time the measurement is carried out. In the Schrödinger picture the operators have rather little prominence, they can be replaced rather completely by transition probabilities.

The same two pictures are possible also in relativistic quantum theory but I wish to submit that one obtains a more simple and unified picture if one looks at the laws of motion as a particular relativistic transformation: as a time displacement. The, Heisenberg's equations of motion for operators (10) are on a par with the equations for the other infinitesimal transformations of relativistic invariance. From this point of view, the equations (11)

$$-i\,\partial q/\partial x^\mu = [P_\mu, q]$$

are four equivalent equations giving the infinitesimal displacements in space and time. Similarly, Schrödinger's equation for the time rate of change of the wave function is one of the transformation equations of the state function under the inhomogeneous Lorentz group; the hermitean nature of Hamilton's operator guarantees the time displacement operators to be unitary. The point of view which I am advocating extends the concepts of Heisenberg and Schrödinger picture to all relativistic transformations—of which the equation of motion is only one—otherwise it does not introduce any new element into the theory.

In spite of the well known and oft discussed equivalence of the Heisenberg and Schrödinger pictures, the three postulates of relativistic invariance present themselves in rather different lights in the two pictures and focus the attention at different aspects of the theory. Let me begin with the Schrödinger picture. Postulate (*b*) when formulated in terms of transition probabilities simply means

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(9) E. SCHRÖDINGER: *Sitzungsber. preuss. Akad. Wiss. Physik.-math. Kl.*, p. 418 (1930).

(10) The concept of time dependent operators was introduced already by M. BORN and P. JORDAN: *Zeits. f. Phys.*, **34**, 858 (1925).

(11) These equations are contained already in the articles of reference (3).

that the transition probability is independent of the frame of reference: if φ and ψ are two states and φ', ψ' their translations, the transition probabilities $\varphi \to \psi$ and $\varphi' \to \psi'$ are equal. One can deduce from this and the distinguishability of all rays in Hilbert space by a purely mathematical argument ([12]) that the translation is effected by a unitary or anti-unitary operator

$$\varphi' = O\varphi; \qquad \psi' = O\psi ;$$

where O is unitary or antiunitary. The operator O depends, of course, on the two coordinate systems between which it effects the translation.

It then follows from postulate (c) that the operation O depends only on the relation of the two frames of reference, not on their absolute position in space-time. From this, one can conclude again by a purely mathematical argument ([13]) that the operators O form, up to a factor, a representation of the group connecting the equivalent frames of reference. This group is the inhomogeneous Lorentz group in ordinary special relativity theory. In the Schrödinger picture, the expression for the relativistic invariance is simple and concise and one is naturally led to consider the simplest sets of state functions which are already relativistically invariant. As is well known—and I shall review the underlying arguments—the set of all possible states of each elementary particle forms such a simplest relativistically invariant set and I shall also review the question which of the properties of these particles can be obtained from the postulates of relativistic invariance. While this part of the consideration of the simplest systems of the Schrödinger picture is quite satisfactory, it must be emphasized that the basic postulate that all self adjoint operators are observable is surely incorrect and it is equally incorrect to assume that all wave functions which differ more than by a multiplicative factor can be distinguished ([8]). This fact casts a shadow on the whole theory as one does not know how far one will have to go in the restriction of the concept of the observable and which of the wave functions represent the same physical system. A second equally great, or perhaps even greater flaw of the Schrödinger picture as here defined is that it does not naturally suggest the consideration of local field observables or anything to substitute for these.

When turning now to the Heisenberg picture, it is well to reemphasize that the actual content of the Heisenberg and Schrödinger pictures in their original forms is the same and that the same physical theories can be formulated in either picture. However, in the Heisenberg picture it is more natural to speak about the translation of observables (rather than wave functions) and to consider such observables which have a meaning in all coordinate systems. Since

([12]) E. P. Wigner: *Gruppentheorie und ihre Anwendungen etc.* (Braunschweig, 1931), Anhang to Chapter XX.

([13]) E. P. Wigner: *Ann. of Math.*, **40**, 149 (1939).

the usual non-relativistic observables refer to an instant of time but to at least a finite portion of space, their set is not translatable: most non-relativistic observables of a coordinate system at rest are not observables for a moving coordinate system. This difficulty is avoided in the Schrödinger picture by first extracting an essence from the observables, the state vector, and translating only the state vector. It is more natural, from the point of view of the Heisenberg representation, to *translate the observables directly* and this is more natural also physically. It implies the consideration of such sets of observables the members of which have a meaning for all frames of reference. Such sets of observables can be obtained from the non-relativistic set in two ways: either by extending that set or by restricting it. If one chooses the second way one is naturally led to observables which are defined at points of space-time, i.e. which are not only instantaneous but also refer to a single point of space. In this way one is led, rather naturally, to the operators of local fields. The other way to make the operators directly translatable between coordinate systems is to permit them to assume a finite extension not only in space but also in time. This possibility of non-local measurements has not been explored very fully. Both points of view are in a certain contrast with that of the Schrödinger picture as I presented it: this does not demand a direct translation of observables but only a translation of a concentrate of the results of observations, of the state vector. The fundamental equivalence of the Heisenberg and Schrödinger pictures manifests itself in the following way. If there are enough observables, the corresponding self adjoint operators will distinguish between any two rays of Hilbert space. Then, two sets of operators which satisfy our postulate (*b*) can be transformed into each other by a unitary similarity transformation $Q \to UQU^{-1}$ or by such a transformation coupled with transition to the conjugate imaginary. This is again a purely mathematical result. If these transformations satisfy postulate (*c*) they again form a representation of the group connecting equivalent observers.

This argument is unquestionably correct but it can be replaced, in local field theories, by a more direct argument which is based on the fact that the field operators have a direct translation into every coordinate system—they transform like scalars, vectors, etc. The equations of motion are the same in each coordinate system and they are invariant if the field quantities are properly transformed. Thus the operators are directly translated and the translation satisfies postulates (*b*) and (*c*). The existence of a similarity transformation UQU^{-1} to transform field quantities from one to another coordinate system would not be necessary if one could really consider the field operators as the only observables. The existence of U is nevertheless usually inferred from the invariance of the commutation relations between fields ([14]). *This*

([14]) Cg. e.g. J. M. JAUCH and F. ROHRLICH: *The Theory of Photons and Electrons* (Cambridge, Mass., 1955), p. 11.

inference is surely not valid; whether it is correct to the extent to which it is meant to be correct, is open to some doubt [15]. Nevertheless, the success of the field theories and simplicity of the preceding argument suggest a more detailed discussion of the problems of these theories.

4. – The Field Theories.

The restricted set of operators which were defined as local field operators do not form a sufficiently large set to distinguish between all rays in Hilbert space. Therefore, if one adopts the restriction that only local fields are observable, one has significantly modified or rather restricted the original theory. In view of the history of theoretical physics in the last ten years, which is a rather unbroken line of successes of local field theories, one is very much tempted to limit the concept of observables to these local fields. As was mentioned before, they form a relativistically invariant set. Furthermore, the limitation of the concept of observables to these is rather natural and excludes in particular all observables which cause trouble with the relativistic invariance and some of which are, in fact, so far removed from ordinary observables that one would not know how to go about their measurement. The superselection rules about which I am going to speak are also an expression for the fact that not all self adjoint operators are observable. The limitation of the observables to fields includes all superselection rules which have been recognized so far [16].

The postulate that only field quantities are observable has not been formulated to my knowledge as explicitly as I just formulated it but must have been present in the minds of many field theoreticians. In particular, FEYNMAN once made a statement which came very close to this. It is, of course, true that several difficulties have to be overcome before one will be fully satisfied that it is reasonable to consider only the field quantities as observables. First among these is the definition of the field after renormalization. Second, it will be necessary to modify the analysis of field measurements, as given by BOHR and ROSENFELD and extended by CORINALDESI [17]. It seems to me

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[15] For the problematical nature of such proofs cf. L. GARDING and A. S. WIGHTMAN: *Proc. Nat. Acad. Sc.*, **40**, 617, 622 (1954) also L. VAN HOVE: *Physica*, **18**, 145 (1952) and R. HAAG: reference [6] and *Kon. Danske Vidensk. Selskab, Math.-fys. Medd.*, **29**, No. 12 (1955).

[16] Cf. reference [8] and L. L. FOLDY: *Phys. Rev.*, **93**, 1395 (1953); S. WATANABE: *Rev. Mod. Phys.*, **27**, 26, 40 (1955).

[17] N. BOHR and L. ROSENFELD: *Kon. Danske Vid. Selskab Mat.-fys. Medd.* **12**, No. 8 (1933); *Phys. Rev.*, **78**, 194 (1950); E. CORINADLESI: *Nuovo Cimento*, **8**, 494 (1951); **9**, 194 (1952). In particulars these articles do not deal fully with the problems raised by O. HALPERN and M. H. JOHNSON (*Phys. Rev.*, **59**, 896 (1941)). Cf. also B. FERRETTI: *Nuovo Cimento*, **12**, 558 (1954).

that in its present form this analysis shows rather the difficulty than the possibility of local field measurements. Third, if one looks at the set of local fields more closely, they form only a limiting case of a relativistically invariant set and there is no set of field quantities the measurement of which would be truly translatable. Thus even in a purely electromagnetic field, if one knows the magnetic field on a space-like surface, all one can say about the electric field, or about the magnetic field in another frame of reference, is that it is most probably infinite. The great successes of the field theories of which we are so proud are mostly in the line of calculations and not in the line of conceptual clarifications. For these reasons it appears doubtful to me that the solution of the problem of observables is as simple as restriction to local field quantities.

5. – Discussion of Elementary Particles by means of the Schrödinger Picture.

It was mentioned before that, in the Schrödinger picture, a unitary or antiunitary transformation corresponds to every relativistic transformation and also that the relativistic transformations include, as a particular case, the equation of motion. It would be erroneous to infer from this that the transformation properties of a system will define all its physical properties. The transformation properties and the equations of motion are only one aspect of the behavior of a physical system. There are many others, such as the configuration of the particles contained therein, which do not affect the transformation properties but remain physically significant. This comes back to the general question of observables and it is clear that not all observables can be uniquely determined on the basis of their transformation properties.

The opposite problem of finding the transformation properties, that is the representation, for a given system, is somewhat easier to solve. If we consider, in particular, an elementary particle, it is natural to postulate that it should not be possible to decompose its states into linear sets which are also relativistically invariant. If such a decomposition were possible, one would call each subset of invariant states the states of a different particle. The same assumption or convention, expressed in terms of the concepts of representation theory, states that the representation which gives the transformation of the states of an elementary particle is irreducible. Irreducible is the mathematical term for the absence of linear sets of states (except the set of all states) which are invariant under the group considered. One is thus led to the conclusion that an irreducible representation of the inhomogeneous Lorentz group—that is the group of Lorentz transformations and displacements in space and time—corresponds to every elementary particle. The representatives can be unitary or antiunitary operators.

The point of view which is outlined above [18] is not quite customary and has often been misunderstood. It is more customary [19] to start with a wave function which, similar to Dirac's wave function [2], contains the variables of space-time and a spin coordinate and find equations which can be transformed relativistically. This more customary procedure has the advantage that it gives a description in which the coordinates are diagonal. It has the disadvantage of proceeding by trial and error. It has the further disadvantage that the same physical situation can be described by a variety of equations and it is not easy, within the framework of the theory, to recognize which equations are equivalent. If one is malicious, one can say that it has the disadvantage of its advantage being illusory because the variables x, y, z, t do not actually correspond to the position of the particle in space-time: actually in many cases, such as that of light quanta, it is not physically meaningful to define the position of the particle [20]. This does not diminish, of course, the merit of the original discovery of invariant equations, such as Proca's.

The standpoint which I am adopting conforms with the view that all relativistic transformations are on the same footing and that the equation of motion is a special case of these. The fact that all representations have infinitely many dimensions is only an expression for the fact that each particle is capable of assuming infinitely many linearly independent states. These are distinguished principally by their momenta, or positions, and considering the state of affairs as it really exists, it would be most embarrassing if any physical system could assume only a finite number of linearly independent states.

The first question which arises in connection with the investigation of the irreducible representations is: to which coordinate transformations correspond unitary and to which antiunitary operators. As far as the elements of the proper inhomogeneous Lorentz group is concerned, the answer is easy to give. All elements of the proper group can be obtained continuously from the unit element (this is what one means by the proper group) and the corresponding transformations must be obtainable continuously from the unit transformation.

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(18) It is contained in various publications such as reference (13) and V. Bargmann and E. P. Wigner: *Proc. Nat. Acad. Sc.*, **34**, 211 (1948); E. Inonu and E. P. Wigner: *Nuovo Cimento*, **9**, 719 (1952); R. Haag: reference (15).

(19) Cf. Al. Proca: *Journ. de Phys.*, **7**, 347 (1936); N. Kemmer: *Proc. Roy. Soc.*, **166**, 127 (1938); **173**, 91 (1939); W. Pauli and M. Fierz: *Helv. Phys. Acta*, **12**, 297 (1939); L. de Broglie: *Compt. Rend.*, **209**, 265 (1939); H. A. Kramers, F. T. Belinfante and T. K. Lubanski: *Physica*, **8**, 597 (1941); H. Bhabha: *Rev. Mod. Phys.*, **17**, 200 (1945); H. Hönl and H. Boerner: *Zeits. f. Naturforsch.*, **5a**, 353 (1950); K. J. Le Couteur: *Proc. Roy. Soc.*, A **202**, 284, 394 (1950); F. L. Bauer: *Ber. Bayer. Akad. d. Wissensch.*, p. 112 (1952); H. Yukawa: *Phys. Rev.*, **91**, 415, 416 (1953).

(20) This was recognized already by P. A. M. Dirac: *Proc. Roy. Soc.*, **114**, 243 (1927). Cf. also T. D. Newton and E. P. Wigner: *Rev. Mod. Phys.*, **21**, 400 (1949).

This is not possible for antiunitary transformations and the operators which correspond to elements of the proper group, that is which do not involve either a space or a time inversion, must be unitary.

Since every transformation of the inhomogeneous Lorentz group is either a member of the proper group, or is a product of a member of the proper group and a space inversion, or a time inversion, or both, it suffices to determine the unitary or antiunitary character of the operations which correspond to inversions. It is important to note then that if there are two quantum mechanical systems in one of which a unitary operator corresponds to a symmetry transformation (such as an inversion), while in the other one the same symmetry transformation is represented by an antiunitary operator, the two systems can not be united to form a common system, even if one excludes interaction, unless all the observables which relate to the joint system are represented by sums of two operators, one of which refers entirely to the first, the second one entirely to the second system. This appears a too drastic restriction of the observables and one must conclude that every transformation is represented either in all physical systems by a unitary transformation, or in all physical systems by an antiunitary transformation. The first is the case for space inversions, the second for time inversion [21].

Once the unitary or antiunitary nature of the relativity transformations is known, the determination of the irreducible representations is again a purely mathematical problem [22]. The results will not be given here in detail. Each representation is characterized by two parameters which will be called, in anticipation of considerations which follow, mass and spin. If the mass is positive, it can assume any value: relativistic invariance alone does not give any indication of possible mass values or their ratios. The spin determines the number of linearly independent states which have the same momentum four vector, except that this number is always 1 or 2 if the mass is zero, that is, if the momentum vectors are on the light cone. Second, contrary to common opinion, the parity of a particle is not determined by its relativistic transformations alone. The same applies for its transformation with respect to time inversion.

The representation is something very abstract and before it can be considered to give a theory of the elementary particle, one must find the observables for the most common physical quantities. This can be done by accepted methods which have, however, little to do with the considerations which led

[21] This point is indicated also by H. Umezawa, S. Kamefuchi and S. Tanaka: *Prog. Theor. Phys.*, **12**, 383 (1955). The same result was obtained by the present writer on the basis of the consideration of time displacements and the positive nature of the energy (*Göttinger Nachr.*, 1932, p. 546).

[22] See reference [13]. It should be noted, however, that the time inversion considered there is Pauli's unitary operator (cf. *Rev. Mod. Phys.*, **13**, 203 (1941)).

to the representation. It may be of some interest, therefore, to point out that the operators for the components of the momentum can be determined, except for a factor, if it is demanded that they be invariant with respect to displacement and transform, under homogeneous Lorentz transformations, as a four vector. The only operators which satisfy this requirement are the infinitesimal operators of displacement (or constant multiples of these)—there is no other quartet of operators in the Hilbert space of the representation which is translation-invariant and a four-vector. This also provides a justification for our having called mass the length of the four vector formed by these infinitesimal operators. NEWTON and I tried to obtain the position operators in a similar way ([22]) and were led to one of the possibilities envisaged already by PAPAPETROU and by MAURICE PRYCE ([23]).

A somewhat more intricate and perhaps also more interesting question is that of the formation and disintegration of semistable compounds. A full answer to this question would be actually much more informative than might appear offhand: all unstable particles, such as mesons, can be considered to be semistable compounds and most reactions between particles can be thought of as proceeding via the intermediate formation of an unstable compound. Similar to the situation with respect to the masses of elementary particles, symmetry considerations permit one to draw no conclusion as to the actual energy of the semistable compounds, they only permit the definition and general characterization of these compounds. The corresponding rules were largely determined already by MICHEL ([24]), they give the characteristics of the compound states which can be formed by the collision of two particles with given spin and mass. The most important ones of these rules we owe to LANDAU and to YANG ([25]), the oldest one is that which states that a $0 \rightarrow 0$ transition with the emission of a light quantum is absolutely forbidden. A 0 spin semistable particle cannot be formed in the collision between a 0 spin particle and a spin 1 particle with zero restmass (light quantum). The results for finite restmass are known from collision theory; if only one of the restmasses is zero, they show a great resemblance to the states of molecules, the spin of the zero restmass particle assuming the role of the electronic angular momentum along the internuclear axis. It should be pointed out that the aforementioned selection rules do not tell the whole story which can be obtained from symmetry arguments: inferences can be drawn concerning the state of

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([23]) A. PAPAPETROU: *Praktika Acad. Athènes*, **14**, 540 (1939); **15**, 404 (1940); M. H. L. PRYCE: *Proc. Roy. Soc.*, **195**, 62 (1948).

([24]) Cf. L. MICHEL: *Report on the 1953 meeting of the IUPAP*, p. 272. This article contains also information on the rules resulting from the symmetries mentioned in reference ([4]).

([25]) L. D. LANDAU: *Dokl. Akad. Nauk USSR*, **60**, 207 (1948); C. N. YANG: *Phys. Rev.*, **77**, 242 (1950).

polarization of the disintegration products of the semistable particle which are equally interesting [26].

I like to illustrate this with an example: the measurement of the parity ratio of two particles with zero spin. At the same time, this will throw some light on the question of the parity i though this question will be discussed, I presume, more in detail in later reports. In order to measure the parity difference between two particles, which we assume for the sake of simplicity to have zero spin, we can let a beam of polarized slow neutrons impinge on the particle with higher restmass. The neutrons can be replaced, of course, by other particles with spin $\frac{1}{2}$. If the incoming particle is sufficiently slow only its s part, i.e. the $l = 0$ part, will react with the spin zero particle. Because of the axial nature of the spin, the collision system will have a plane of symmetry in the direction perpendicular to the direction of polarization of the incoming neutron. The neutrons will then react with the spin zero particle and will transform it, with a certain probability, into the spin zero particle with the smaller restmass. The mass difference between the two spin zero particles will increase the kinetic energy of the neutrons so that a certain fraction of the outgoing neutron beam will have a higher kinetic energy than the ingoing beam and will be distinguishable therefrom. If there is no parity difference between the two spin zero particles, the outgoing wave will also be an s wave, i.e. will have $l = 0$, and the direction of the spin of the neutrons will not have changed. The two spin components of the outgoing wave will be

$$\psi_{\text{spin as originally}} = \frac{1}{r} \exp[ikr] \quad \text{and} \quad \psi_{\text{spin flipped}} = 0 .$$

If the parities of the two spin zero particles are different, the outgoing neutron wave with the higher kinetic energy will be a p-wave, i.e. will have $l = 1$, and its two spin components will be, in the plane perpendicular to the direction of polarization of the incoming neutron, asymptotically

$$0 \quad \text{and} \quad \exp[i\varphi] \frac{1}{r} \exp[ikr] .$$

It will be possible to observe the parity difference by observing the direction of the spin of the outgoing neutron in the plane perpendicular to the original direction of polarization: if there is no flip in the spin direction, the two parities

(26) Some results for the angular distribution and polarisation were obtained already by C. L. CRITCHFIELD and E. TELLER: *Phys. Rev.*, **60**, 10 (1941); D. R. HAMILTON: *Phys. Rev.*, **71**, 546 (1947); E. EISNER and R. G. SACHS: *Phys. Rev.*, **72**, 680 (1947); L. WOLFENSTEIN and R. G. SACHS: *Phys. Rev.*, **73**, 528 (1948); C. N. YANG: *Phys. Rev.*, **74**, 764 (1948).

are the same, if there is a flip, the parities are opposite. This is a quantum mechanical measurement in the best orthodox sense (7): corresponding to the two possible values of the quantity to be measured, there are two possible outcomes of the measurement. I brought this example up to illustrate further relations based solely on symmetry considerations which go beyond the relations of Table I.

TABLE I. – *Semistable Particles resulting from the Collision of two Particles.*
Two identical particles (*).

$s_1 = s_2 = 0 \to J = 0, 2, 4, \ldots$ with even parity. No states with odd parity.
$s_1 = s_2 = \frac{1}{2} \to J = 0, 2, 4, \ldots$ with even parity. All J with odd parity.
$s_1 = s_2 \geqslant 1$; $m > 0 \to$ All J with both even and odd parities.
$s_1 = s_2 \geqslant 1$; $m = 0 \to \begin{cases} J = 0, 2, 4, \ldots \text{ with both even and odd parities.} \\ J = 2s,\ 2s+1, \ldots \text{ with parity } (-)^{2s}. \end{cases}$

Two different particles with finite restmass.

$s_1 = s_2 = 0 \to \begin{cases} J = 0, 2, 4, \ldots \text{ with even parity.} \\ J = 1, 3, 5, \ldots \text{ with odd parity.} \end{cases}$
$s_1 = 0$; $s_2 \geqslant 1 \to$ All J with both parities, except $J = 0$ only with parity $\omega = (-)^{s_2}\omega_1\omega_2$.
s_1, $s_2 \geqslant \frac{1}{2} \to$ All J with both parities.

First particle with finite, second with zero restmass.

$s_2 \leqslant \frac{1}{2} \to$ same as two different particles with finite restmass.
$s_1 = 0$, $s_2 \geqslant 1 \to J = s_2,\ s_2 + 1,\ s_2 + 2, \ldots$ with both parities.
$s_1 \geqslant s_2 \geqslant 1 \to$ All J with both parities.
$s_2 > s_1 \geqslant \frac{1}{2} \to J = s_1 - s_1,\ s_2 - s_1 + 1,\ s_2 - s_1 + 2, \ldots$ with both parities.

Two different particles with zero restmass.

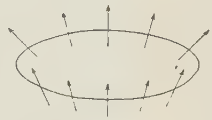
$s_1 = s_2 = 0 \to$ same as two different particles with finite restmass.
otherwise $\to J = |s_1 - s_2|,\ |s_1 - s_2| + 1,\ |s_1 - s_2| + 2, \ldots$ with both parities.

(*) The particles with integer and half integer spin were assumed to have symmetric and antisymmetric wave functions, respectively, and the parities were assumed to be ± 1 and comparable throughout; s_1 and s_2 are the spins of the colliding particles. The case that the mass m of the identical particles is 0 is included under this heading.

The experiment which I mentioned also illustrates the point that if the parity relation is measurable it can have only two values: equal and opposite. In particular, any linear combination of the wave functions which correspond to the two cases, such as one with the two components.

$$\alpha \frac{1}{r} \exp[ikr] \qquad \text{and} \qquad \beta \exp[i\varphi] \frac{1}{r} \exp[ikr]\,,$$

would yield a spin direction of the outgoing neutron in the plane perpendicular to the polarization of the incoming neutron as illustrated. Such a distribution of the directions of polarization will not permit a symmetry plane perpendicular to the original direction of polarization and would be in direct conflict with the symmetry principle which establishes the concept of parity.



The preceding consideration can be generalized to show that if the parity difference between two particles can be measured, it can yield only the two results « equal » or « opposite ». Can we conclude from this that only two parities exist? In our opinion (8), the answer to this question is « no », because it is quite possible that none of the measurements will yield any result. It would mean, for the preceding example, that the neutrons simply do not transform the spin zero particles into each other so that there is no outgoing beam on which to measure the spin. If there is such a block against measuring the spin we say that a superselection rule separates the two types of particles. The qualification « super » is added, not only for nationalistic reasons, but also to indicate that not only is a spontaneous transition impossible but that a transition cannot be induced even by measurements.

The preceding consideration, and the generalization to which I alluded, shows that all measurements of parity changes can give either one of the results: same parity, opposite parity, or give no result. It is possible that the matter simply ends here and that it is not reasonable at all to define a parity difference between two particles. In my opinion this is the case, for instance, as far as protons and electrons (or positrons) are concerned. However, it should be noted that the preceding consideration refers only to the parity change between one particle of each kind. Even if it should be, in principle, impossible to define such a parity difference, it is quite conceivable that the parity of a *pair* of particles A can be compared with the parity of particle B. In this case—and this is a certain change of point of view on my part—it might be desirable to define a parity i or i, or similar directly not measurable parities.

6. – Conclusion.

The question of symmetry, in particular, relativistic symmetry in quantum theory, is one in which not only I but several of my colleagues are deeply interested (27). The point of view which I adopted was that our knowledge

(27) My indebtedness is particularly great to Drs. V. BARGMANN and A. S. WIGHTMAN.

of the interaction of particles is not yet adequate to read off the symmetry properties from the Lagrangians which we use. Rather, the symmetry properties should be a guide in establishing the proper physical picture. The element which was emphasized particularly on the basis of this point of view is that the conclusions at which one can arrive on the basis of symmetry considerations depend very much on the type of measurements which are possible « in principle ». The situation is most similar, perhaps, to that in thermodynamics in which, also, many results depend on the type of equipment that one can use « in principle » to construct a perpetuum mobile. The more liberal one is in permitting in practice unrealizable equipment to establish a perpetuum mobile, the more the scope of thermodynamics is increased, the greater the number of phenomena which we can treat with it. Similarly, the symmetry considerations will have greatest scope if we permit most « measurements » as possible in principle, if we consider it permissible to assume any type of interaction with the measuring equipment. However, we know that there is a limitation in this regard, that it is, for instance, not permissible to assume that every self adjoint operator in the Hilbert space of field theories is measurable. The exact limitation is not known at present and it will be an intriguing problem to explore it.

RIASSUNTO (*)

Viene eseguita una particolareggiata analisi sulle possibilità teoriche per la costruzione di una meccanica quantistica e quindi di una descrizione delle particelle elementari, fondata su una definizione dei postulati di invarianza relativistica e sulle proprietà di simmetria connesse col gruppo completo di Lorentz, comprese traslazioni nello spazio-tempo e inversioni sia delle coordinate spaziali che della coordinata temporale. È poi messo in luce come la validità di ogni conclusione tratta da considerazioni di simmetria sia condizionata dal problema fondamentale della misurabilità delle grandezze di campo.

(*) *A cura della Redazione.*

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Quantization of Non-Linear Fields.

Ken-iti Gotō

Osaka University - Osaka, Japan

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Summary. — A method of developing quantized non-linear field theories is proposed in the functional representation, where the non-linearity of the classical equation appears as the peculiar form of field operators and the field equation for state vectors becomes a linear functional differential equation. The method to diagonalize the field Hamiltonian is illustrated by the free electromagnetic field, the hydrodynamical field and the non-linear meson field.

1. – Introduction.

The quantization of linear wave fields, without interaction, can be performed and the energy eigenvalues can be obtained without difficulties. However any satisfactory method of non linear field quantization is not yet given successfully. Although methods of introducing reasonable commutation rules are given by several authors so as to hold whether the field equations are linear or non linear, any method of diagonalization of the Hamiltonian for quantized non-linear fields is not yet given in success. Only in the case where the non-linear term arises from the interaction of otherwise linear fields and has a small coefficient, we can give approximate diagonalizations by perturbation methods. But in the case of a large non-linear term we have no means of treating the quantized fields in the continuous space. Only in a discontinuous lattice space such methods have been developed by Wentzel (1), Yennie (2),

(1) G. Wentzel: *Helv. Phys. Acta*, **13**, 269 (1940).
(2) D. R. Yennie: *Phys. Rev.*, **88**, 527 (1952).

Rosenstock [3], Finkelstein [4], Schiff [5] and Holland [6], but we cannot go to the continuous case in this method. Other attempts to quantize non-linear fields are also given in several ways following the ordinary methods which were successful for the linear fields. For example a method to quantize the hydrodynamical field equation is developed by Landau [7], Bogoljubov [8], Kronig [9], Thellung [10], Ito [11], Nishiyama [12], Ziman [13] and Tyabji [14], but these methods use the Fourier expansion of operators and the quantization is concerned essentially with the linear terms and the non-linear terms are not taken into account in earnest. Recently a new method of quantized non-linear field theory was proposed by Heisenberg [15,27] in the scheme of perturbation expansion and the propagator, but we can not follow it with the mathematical rigor in the present stage, even if it involves excellent ideas and instructive considerations and so it may be taken into account with rigor in the future. Finally, Born's [16] non-linear field theory, which was proposed for the first time as an essentially non-linear theory and developed by Infeld [17], Hofmann [18], Weyl [19], Pryce [20], Weiss [21], Husimi [22] and Imai [23], was a unified theory but we can not derive reasonable solutions from the quantized theory as yet.

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(3) H. B. Rosenstock: *Phys. Rev.*, **93**, 331 (1954).

(4) R. J. Finkelstein; *Phys. Rev.*, **75**, 1079 (1949).

(5) L. I. Schiff: *Phys. Rev.*, **92**, 766 (1953).

(6) D. H. Holland: *Phys. Rev.*, **98**, 788 (1955).

(7) L. D. Landau: *Journ. Phys.*, **6**, 71 (1941).

(8) N. N. Bogoljubov: *Journ. Phys.*, **11**, 23 (1947).

(9) R. Kronig and A. Thellung: *Physica*, **18**, 749 (1952).

(10) A. Thellung: *Physica*, **19**, 217 (1953).

(11) H. Itō: *Prog. Theor. Phys.*, **9**, 117 (1953); **13**, 543 (1955).

(12) T. Nisihiyama: *Prog. Theor. Phys.*, **8**, 655 (1952).

(13) J. M. Ziman: *Proc. Roy. Soc.*, **219**, 257 (1953).

(14) S. F. B. Tyabji: *Proc. Camb. Phil. Soc.*, **50**, 449 (1954).

(15) W. Heisenberg: *Nachr. Wiss. Göttingen*, **8**, 111 (1953); *Zeits. f. Naturf.*, **9a** (1954).

(16) M. Born: *Proc. Roy. Soc.*, **143**, 410 (1934).

(17) M. Born and L. Infeld: *Proc. Roy. Soc.*, **144**, 425 (1934); **147**, 522 (1934).

(18) B. Hofmann and L. Infeld: *Phys. Rev.*, **51**, 765 (1937).

(19) H. Weyl: *Phys. Rev.*, **46**, 505 (1934).

(20) M. H. L. Pryce: *Proc. Roy. Soc.*, **150**, 166 (1935).

(21) P. Weiss: *Proc. Roy. Soc.*, **169**, 102 (1939); **169**, 119 (1939).

(22) K. Husimi: *On the Variational Principle in the Field Theory* (in Japanese, 1944).

(23) I. Imai: *Prog. Theor. Phys.*, **2**, 97 (1947); a lecture at the Yukawa Hall, Kyoto University (Jan. 1955).

(24) Friedrichs: *Mathematical Aspect of Quantum Theory of Fields* (1953).

(25) S. F. Edwards and R. E. Peierls: *Proc. Roy. Soc.*, **224**, 24 (1954); S. F. Edwards: *Proc. Roy. Soc.*, **228**, 411 (1955); preprints (ps meson case).

(26) B. Davison: *Proc. Roy. Soc.*, **225**, 252 (1954).

(27) K. Symanzik: *Zeits. f. Naturf.*, **9a**, 908 (1954).

In this paper we will propose a method of developing quantized non-linear field theory by using the functional representation and the functional space formulation, which was developed by FRIEDRICHS [24], EDWARD-PEIERLS [25], DAVISON [26], SYMANZIK [27], IWATA [28] and COESTER [29], and further by introducing new functional differentiations. We give up the classical field equation between the field operators and establish the corresponding equation for the state vectors. In the functional representation this equation becomes a linear functional differential equation. By establishing such generalized Schrödinger equation a formal diagonalization of the Hamiltonian is achieved and by solving it under the boundary condition of the square integrability the energy spectrum is obtained. Illustrations of the method are given with the free electromagnetic field, the hydrodynamical field and the non-linear meson field of Schiff's type. For the quanta of the quantized hydrodynamical field the energy spectrum has a tendency similar to those expected by LANDAU for the interpretation of He II. In the quantization of the non-linear meson field of Schiff's type a relation is given with the method of the lattice space quantization and its quanta have the spectra similar to that of the anharmonic oscillator.

I. General Formulation.

2. – Formulation in the functional representation.

In the usual theory, the classical field equation is derived from a Lagrangian function $L(\varphi^\alpha, \varphi^\alpha_{(\mu)}, \ldots)$, $\varphi^\alpha_{(\mu)} = \partial_\mu \varphi^\alpha$. From the variational principle $\delta \overline{L} = 0$, the field equation is obtained as the Euler equation

$$F(\varphi^\alpha, \varphi^\alpha_{(\mu)}, \ldots) = 0 \,. \tag{2.1}$$

The quantization is performed by introducing the conjugate momenta $\pi^\alpha = \partial L / \partial \varphi^\alpha_{(4)}$ and by establishing the commutation relations

$$\left\{\begin{aligned} &[\varphi^\alpha(\bar{x}, x_4), \varphi^\beta(\bar{x}', x_4)] = 0 \,, \\ &[\pi_\alpha(\bar{x}, x_4), \pi_\beta(\bar{x}', x_4)] = 0 \,, \\ &[\pi_\alpha(\bar{x}, x_4), \varphi^\beta(\bar{x}', x_4)] = i\hbar\, \delta^\beta_\alpha \delta(\bar{x} - \bar{x}') \,, \\ &\bar{x} = (x_1, x_2, x_3) \,, \quad \bar{x}' = (x_1', x_2', x_3') \,. \end{aligned}\right. \tag{2.2}$$

(28) G. IWATA: *Prog. Theor. Phys.*, 11, 537 (1954).
(29) F. COESTER: *Phys. Rev.*, 95, 1318 (1954).

When the field equation (2.1) is linear, we can obtain the general solution or a complete set of orthonormal solutions and we can solve the problems of the quantized field by regarding the coefficients of orthogonal solution-expansions as operators or by introducing the so-called propagation function from the general solution and the appropriate initial condition.

In the non-linear case these methods fail out in general. So we will develop the method of the functional for this case. In the linear case this method also can be developed easily, for we can easily introduce the functional Schrödinger equation by expressing as algebraic forms of π_α and φ^α. Anyhow when the quantization is performed according to (2.2), the commutation relations of $\varphi^\alpha_{(\mu)}$, $\pi^\alpha_{(\mu)}$, ... will become

$$
(2.3)\quad \begin{cases}
[\varphi^\alpha(\bar{x}, x_4), \varphi^\beta_{(l)}(\bar{x}', x_4)] = 0 & l = 1, 2, 3 \\
[\varphi^\alpha_{(l)}(\bar{x}, x_4), \varphi^\beta_{(k)}(\bar{x}', x_4)] = 0 & k = 1, 2, 3 \\
[\pi^\alpha(\bar{x}, x_4), \pi^\beta_{(l)}(\bar{x}', x_4)] = 0 & \\
[\pi_\alpha(\bar{x}, x_4), \varphi^\beta_{(l)}(\bar{x}', x_4)] = -i\hbar\,\delta^\beta_\alpha\,\delta^{(l)}(\bar{x}-\bar{x}'), \quad \delta^{(l)}(x) = \partial\delta(x)/\delta x_l & \\
[\pi_{\alpha(l)}(\bar{x}, x_4), \varphi^\beta(\bar{x}', x_4)] = \quad i\hbar\,\delta^\beta_\alpha\,\delta^{(l)}(\bar{x}-\bar{x}'), & \\
[\pi_{\alpha(l)}(\bar{x}, x_4), \varphi^\beta_{(k)}(\bar{x}', x_4)] = -i\hbar\,\delta^\beta_\alpha\,\delta^{(l)(k)}(\bar{x}-\bar{x}'), \quad \delta^{(l)(k)} = \partial^2\delta(x)/\partial x_l\,\partial x_k, & \\
\cdots\cdots\cdots\cdots\cdots\cdots &
\end{cases}
$$

which are derived from (2.2) by formal differentiation. This is obvious for the linear case where the expansion method can be used properly. But in general this must be verified. Now we postulate the commutation relations (2.2) (2.3) and the field equation of the type

$$
(2.4)\qquad F(\varphi^\alpha, \varphi^\alpha_{(l)}, \ldots, \pi^\alpha, \pi^\alpha_{(l)}, \ldots)\Psi = 0
$$

by rewriting as

$$
(2.5)\qquad F(\varphi^\alpha, \varphi^\alpha_{(\mu)}, \ldots) = F(\varphi^\alpha, \varphi^\alpha_{(l)}, \ldots, \pi^\alpha, \pi^\alpha_{(l)}, \ldots)
$$

and by giving up the equation of the type (2.1). In the functional representation we can put

$$
(2.6)\qquad \begin{cases}
\pi^\alpha = i\hbar\,\dfrac{\delta}{\delta\varphi_\alpha} \\
\pi^\alpha_{(l)} = i\hbar\,\dfrac{\delta^{(l)}}{\delta^{(l)}\varphi_\alpha} \\
\cdots\cdots\cdots
\end{cases}
$$

where $\delta/\delta\varphi^\alpha$ is the ordinary functional derivative [24]

$$\int \frac{\delta\Psi\{\varphi^\alpha(x)\}}{\delta\varphi^\alpha(x)}\,\gamma(x)\,\mathrm{d}x = \lim_{\varepsilon\to 0}\frac{\Psi\{\varphi^\alpha(x)+\varepsilon\gamma(x)\}-\Psi\{\varphi^\alpha(x)\}}{\varepsilon} \tag{2.7}$$

and $\delta^{(l)}/\delta^{(l)}\varphi^\alpha$ is a newly introduced derivative

$$\int \frac{\delta^{(l)}\Psi\{\varphi^\alpha(x)\}}{\delta^{(l)}\varphi^\alpha(x)}\,\gamma(x)\,\mathrm{d}x = \lim_{\varepsilon\to 0}\frac{\Psi\{\varphi^\alpha(x)-\varepsilon\,\partial_l\gamma(x)\}-\Psi\{\varphi^\alpha(x)\}}{\varepsilon}\,. \tag{2.8}$$

In the representation (2.6) the commutation relations (2.2) (2.3) hold automatically, because we can show the relation

$$\left[\frac{\delta^{(l)}}{\delta^{(l)}\varphi^\alpha(x)},\,\varphi^\alpha(x')\right] = \delta^{(l)}(x-x') \tag{2.9}$$

in the same way as the ordinary case deriving the relation [24]

$$\left[\frac{\delta}{\delta\varphi^\alpha(x)},\,\varphi^\alpha(x')\right] = \delta(x-x')\,. \tag{2.10}$$

We can also introduce the operator

$$\int \frac{\delta^{(k)(l)\dots}\Psi\{\varphi^\alpha(x)\}}{\delta^{(k)(l)\dots}\varphi^\alpha(x)}\,\gamma(x)\,\mathrm{d}x = \lim_{\varepsilon\to 0}\frac{\Psi\{\varphi^\alpha(x)+\varepsilon((-\partial_k)(-\partial_l)\dots\gamma(x))\}-\Psi\{\varphi^\alpha(x)\}}{\varepsilon} \tag{2.11}$$

which satisfies the commutation relation

$$\left[\frac{\delta^{(k)(l)\dots}}{\delta^{(k)(l)\dots}\varphi^\alpha(x)},\,\varphi^\alpha(x')\right] = \delta^{(k)(l)\dots}(x-x') \qquad \delta^{(k)(l)\dots}(x) = \partial_k\partial_l\dots\delta(x). \tag{2.12}$$

These operators take simple forms after the functional Fourier transformation.

In order to diagonalize the total Hamiltonian we establish the general functional Schrödinger equation

$$H\left(\varphi^\alpha,\,\varphi^\alpha_{(l)},\,\dots,\,i\hbar\frac{\delta}{\delta\varphi^\alpha},\,i\hbar\frac{\delta^{(l)}}{\delta^{(l)}\varphi^\alpha},\,\dots\right)\Psi(\varphi^\alpha) = \varepsilon\Psi(\varphi^\alpha)\,. \tag{2.13}$$

The boundary condition is that Ψ should be square integrable in the Hilbert space at least in the Friedrichs' meaning. When the state vector Ψ does not depend on the space coordinate x, this becomes the ordinary type equation

$$\overline{H}\Psi = E\Psi\,, \qquad E = \int \varepsilon(x)\,\mathrm{d}x\,, \tag{2.14}$$

and $\varepsilon(x)$ can be interpreted as the energy density at the point x. If we perform the functional Fourier transformation ([25,27]) by

$$\begin{cases} \Psi(\varphi^\alpha(x)) \to \int \widetilde{\Psi}(\widetilde{\chi}^\alpha(k)) \exp\left[i\int \widetilde{\chi}^\alpha(k')\widetilde{\varphi}^\alpha(k')\,\mathrm{d}k'\right]\mathrm{d}\widetilde{\chi}^\alpha(k)\,, \\ \varphi^\alpha(x) \to \widetilde{\varphi}^\alpha(k')\,, \qquad \chi^\alpha(x) \to \widetilde{\chi}^\alpha(k')\,, \end{cases} \tag{2.15}$$

the two sides of the Schrödinger equation become

$$\widetilde{H}\left(i\frac{\delta}{\delta\widetilde{\chi}}, i\frac{\delta^{(l)}}{\delta^{(l)}\widetilde{\chi}}, \ldots, \widetilde{\chi}^\alpha, -\hbar k_l\widetilde{\chi}^\alpha, \ldots\right)\widetilde{\Psi}(\widetilde{\chi}^\alpha) = \widetilde{\varepsilon}(k)\widetilde{\Psi}(\widetilde{\chi}^\alpha)\,. \tag{2.16}$$

When the state vector Ψ does not depend on the coordinate the total energy becomes

$$E = \int \widetilde{\varepsilon}(k)\,\mathrm{d}k\,, \tag{2.17}$$

so $\widetilde{\varepsilon}(k)$ can be interpreted as the energy spectrum. The equation of type (2.11) was established by SCHIFF for the non-linear meson field of Schiff's type and his treatment of the momentum term can be justified by our formulation ([5]).

Our functional equation is a linear functional equation with respect to the differentiations $\delta/\delta\varphi$, $\delta^{(l)}/\delta^{(l)}\varphi$, ...: But it is not easy to solve such an equation directly. So we must use the method of the functional Fourier transformation for the present. Such a transformation involves a functional integration. Integrations over the Hilbert space were treated by WIENER ([31]) and FRIEDRICHS ([24]) in their unique forms, even if any general integral measure in the Hilbert space is not yet established in completely universal manner. However the formal functional Fourier transformation has been used sometimes successfully in physics ([25,26]). Indeed for the sake of the utility of the functional method in physics such a transformation must exist rigorously for, so to speak, temperate functionals, such as the Hermite functionals, for which it exist at least in the Friedrichs' meaning. Here we will use the functional Fourier transformation together with the ordinary Fourier transformation of the coordinates. We assume that such a calculus holds also for non commutative functions. After the transformation

$$\begin{cases} \varphi(x) \to \widetilde{\varphi}(k)\,, \quad \varphi^*(x) \to \widetilde{\varphi}^*(k) \quad [\varphi^*(x), \varphi(x')] = i\delta(x-x')\,, \\ \Psi(\varphi(x)) \to \int \widetilde{\Psi}(\widetilde{\chi}(k)) \exp\left[i\int \widetilde{\chi}(k')\widetilde{\varphi}(k')\,\mathrm{d}k'\right]\mathrm{d}\widetilde{\chi}(k) \end{cases} \tag{2.17}$$

([30]) H. BATEMANN: *Partial Differential Equations of Mathematical Physics* (1932).
([31]) PALEY and F. M. WIENER: *Fourier Transforms in the Complex Domain* (1934).

the functions $\varphi(x)$, $\varphi^*(x)$ are expressed by

$$\varphi(x) \sim \tilde{\chi}^*(k)\,, \qquad \varphi^*(x) \sim \tilde{\chi}(k)\,, \tag{2.18}$$

$$[\tilde{\chi}^*(k),\, \tilde{\chi}(k')] = i\delta(k-k')\,, \tag{2.19}$$

in the $\tilde{\chi}$ representation. The derivatives $\partial_l\varphi(x)$, $\partial_l\varphi^*(x)$ are expressed by

$$\partial_l\varphi(x) \sim ik_l\tilde{\chi}^*(k)\,, \qquad \partial_l\varphi^*(x) \sim ik_l\tilde{\chi}(k)\,. \tag{2.20}$$

The products in the original representation are transfered into the same kind of products in the transformed representation at least for some restricted cases (see the Appendix). Namely the ordinary products become the convolution products after the coordinate Fourier transformation and they restore the original rule as the ordinary products after the functional Fourier transformation by virtue of the δ-type commutation relation (2.19). In the case where φ is diagonal, $\varphi^* = i\delta/\delta\varphi$ and they are expressed in the $\tilde{\chi}$ representation by

$$\varphi(x) \sim i\,\frac{\delta}{\delta\tilde{\chi}(k)}\,, \qquad i\,\frac{\delta}{\delta\varphi(x)} \sim \tilde{\chi}(k)\,. \tag{2.21}$$

$$\partial_l\varphi(x) \sim -\,k_l\,\frac{\delta}{\delta\tilde{\chi}(k)}\,, \qquad i\,\frac{\partial^{(l)}}{\delta^{(l)}\varphi(x)} \sim ik_l\tilde{\chi}(k)\,. \tag{2.22}$$

3. – An illustration by a linear field. The free electromagnetic field.

In order to illustrate the method let us treat the free electromagnetic field as a simple and important example of the linear case. The electromagnetic field in the vacuum is described by

$$\Box \boldsymbol{A} = 0: \tag{3.1}$$

$$\operatorname{div} \boldsymbol{A} = 0, \tag{3.2}$$

for in this case we can take the gauge where $A_4 = 0$. The Hamiltonian is

$$\overline{H} = \frac{1}{2}\int(\boldsymbol{E}(x)^2 + \boldsymbol{H}(x)^2)\,\mathrm{d}\bar{x}\,, \tag{3.3a}$$

which can be written in the form

$$\overline{H} = \frac{1}{2}\int\{E_lE_l + (\partial_kA_m)(\partial_kA_m)\}\mathrm{d}\bar{x} \tag{3.3b}$$

by virtue of (3.2). The quantization is performed as follows,

$$
(3.4) \qquad \begin{cases} [A_k(\bar{x}, x_4),\ A_l(\bar{x}', x_4)] = 0 \\ [E_k(\bar{x}, x_4),\ E_l(\bar{x}', x_4)] = 0 \\ [E_k(\bar{x}, x_4),\ A_l(\bar{x}', x_4)] = i\hbar\,\delta_{kl}\,\delta(\bar{x}-\bar{x}')\,. \end{cases}
$$

In the functional representation where E_k is diagonal we can put

$$
(3.5) \qquad \partial_k A_m = i\hbar\,\frac{\delta^{(k)}}{\delta^{(k)}E_m}\,,
$$

for the commutation relation

$$
(3.6) \qquad [E_l(\bar{x}, x_4),\ \partial_k A_m(\bar{x}', x_4)] = -\,i\hbar\,\delta_{lm}\,\delta^{(k)}(\bar{x}-\bar{x}')
$$

holds. Thus the functional Schrödinger equation (2.11) becomes

$$
(3.7) \qquad \frac{1}{2}\left\{E_l E_l + \left(i\hbar\,\frac{\delta^{(k)}}{\delta^{(k)}E_m}\right)\left(i\hbar\,\frac{\delta^{(k)}}{\delta_m E^{(k)}}\right)\right\}\Psi(E_j) = \varepsilon(x)\Psi(E_j)\,.
$$

After the Fourier transformation

$$
(3.8) \qquad \Psi(E_j(x)) = \int \widetilde{\Psi}(\widetilde{\chi}_j(k))\,\exp\left[i\int\widetilde{\chi}_j(k')\widetilde{E}_l(k')\,\mathrm{d}k'\right]\mathrm{d}\widetilde{\chi}(k)\,\exp\left[-\,i2kx\right]\mathrm{d}k\,,
$$

the transformed Schrödinger equation (2.14) becomes, for even or odd $\widetilde{E}_j$,

$$
(3.9) \qquad \frac{1}{2}\sum\left\{-\frac{\delta^2}{\delta\widetilde{\chi}_l^2} + \hbar^2 k^2 \widetilde{\chi}_l^2\right\}\widetilde{\Psi}(\widetilde{\chi}_l) = \widetilde{\varepsilon}(k)\widetilde{\Psi}(\widetilde{\chi}_l)\,.
$$

Both sides of this equation can be put in the form

$$
(3.10) \qquad \frac{1}{2}\,\hbar k\sum\left\{-\frac{\delta^2}{\delta\chi_l'^2} + \widetilde{\chi}_l'^2\right\} = \widetilde{\Psi}'(\widetilde{\chi}_l') = \widetilde{\varepsilon}(k)\,\widetilde{\Psi}(\chi_l')
$$

by the transformation

$$
(3.11) \qquad \widetilde{\chi}_l = \widetilde{\chi}_l'/\sqrt{k\hbar}\,.
$$

This functional equation is satisfied by the Hermite functional $J_{n_1}\{\widetilde{\chi}_1'\}J_{n_2}\{\widetilde{\chi}_2'\}J_{n_3}\{\widetilde{\chi}_3'\}$ constructed in the method similar to the Friedrichs' one by using the ordinary definition of the Hermite polynomial and the Hermite function familiar to

physicists [32]. We will denote this functional with J_n and the one introduced by FRIEDRICHS with J'_n. The functional J_n satisfies the functional equation

$$\left\{-\frac{\delta^2}{\delta\xi^2}+\xi^2\right\}J_n\{\xi\}=(2n+1)J_n\{\xi\}\,. \tag{3.12}$$

These functionals forms a complete orthogonal set in the space of the square integrable functionals in the meaning of FRIEDRICHS [24]. Thus we have the energy spectrum

$$\varepsilon(k)=\hbar k(n+\tfrac{3}{2})\,,\qquad n=n_1+n_2+n_3 \tag{3.13}$$

this coincides with the result of the ordinary method of quantization except for the zero point energy. This arises from the treatment postulating the functional equation of the type (2.4) instead of the type (2.1) and regarding the three freedoms of the coordinates as independent, to the contrary to the ordinary method treating the only one freedom of the frequencies. These circumstances are similar to the case of the harmonic oscillator in the three dimensions with the zero point energy $\frac{3}{2}$ and that in the one dimension with the zero point energy $\frac{1}{2}$. If we neglect these zero point energies as unphysical for the present, our method of quantization will give physically meaningful results as well as the ordinary method. The fact that the restriction to the even and odd functions can produce the experimentally justified result, namely the correct energy spectrum, seems to mean that the photon corresponds mainly to such symmetrical excitation of the vacuum. Indeed the state vectors corresponding to such restricted states are sufficient to express those for arbitrary states as certain linear combinations, for the Hermite functionals form a complete set. If $E(x)$ is a plane wave, $E(k)$ is always even.

In the case of the linear field, if we restrict φ's to plane waves we can easily express $\delta^{(n)}/\delta^{(n)}\varphi$ with $\delta/\delta\varphi$ and can solve the functional equations of the type (2.13) or (3.7) without the Fourier transformations. But this will not give so good approximations for non linear fields.

II. Hydrodynamical Field.

4. – The quantum hydrodynamics.

The problem of the quantum hydrodynamics or to quantize the hydrodynamical field was attempted by LANDAU for the first time in order to explain the curious properties of HeII in the superfluid state. After that many authors

[32] SZEGÖ: *Orthogonal Polynomials* (1939).

tried to establish the theory on a more reasonable and firm ground. But these methods of formulation involve unreasonable features from the field theoretical point of view, because the methods suitable for only linear fields are used for the essentially non-linear field of the hydrodynamics. For example the treatment of the phonon state concerning the small linear vibration of the field is reasonable but that of the roton state concerning the non-linear rotational motion is not so. Particularly the lack of the simultaneous diagonalization of respective terms in the Hamiltonian seems to be the most undesirable feature from the physical point of view. Indeed the inadequency of the occupation number representation for the diagonalization of some kinds of non-linear field Hamiltonians lies in the non-diagonalizability of odd powers of creation and annihilation operators. Thus we will treat, here, this problem with our functional method.

The equation of the density (ϱ) – velocity (v) field of the rotational non-viscous fluid can be derived from a Lagrangian and can be put into the Hamiltonian formalism ([13,30]). Introducing the velocity potential of Clebsch φ, ψ and σ the Hamiltonian density is written as

(4.1) $$H = \tfrac{1}{2}\varrho \boldsymbol{v}^2 + V(\varrho)$$

(4.2) $$= \tfrac{1}{2}\varrho\nabla\varphi\nabla\varphi + \nabla\varphi\sigma\nabla\psi + \tfrac{1}{2}\varrho^{-1}\sigma\nabla\psi\sigma\nabla\psi + V(\varrho)$$

(4.3) $$V(\varrho) = \varrho\int_{\varrho}^{\varrho_0}\frac{p-p_0}{\varrho^2}\,\mathrm{d}\varrho = $$
$$= \frac{1}{2}\,\frac{c_0^2}{\varrho_0}(\varrho-\varrho_0)^2 + \frac{1}{3\,!}\left[\frac{\mathrm{d}}{\mathrm{d}\varrho}\left(\frac{c^2}{\varrho}\right)\right]_0(\varrho-\varrho_0)^3 + \dots \qquad \left(c^2 = \frac{\mathrm{d}p}{\mathrm{d}\varrho}\right)$$

and the conjugate momenta π_φ, π_ψ are determined ([18]) as

(4.4) $$\pi_\varphi = \varrho\,, \qquad \pi_\psi = \sigma\,.$$

The quantization is performed obviously by

(4.5) $$[\varphi(\bar{x}, x_4), \varrho(\bar{x}', x_4)] = i\hbar\,\delta(\bar{x}-\bar{x}') \qquad [\psi(\bar{x}, x_4), \sigma(\bar{x}', x_4)] = i\hbar\,\delta(\bar{x}-x')\,,$$

other commutators vanishing. From these commutation relations we can derive those which were assumed by Landau originally

(4.6) $$[v_l(\bar{x}, x_4), \varrho(\bar{x}', x_4)] = -i\hbar\,\delta^{(l)}(\bar{x}-\bar{x}')\,,$$

(4.7) $$[v_j(\bar{x}, x_4), v_k(\bar{x}', x_4)] = -i\hbar\,\delta(\bar{x}-\bar{x}')\omega_l\,,$$

(4.8) $$\omega_l = \partial_j v_k - \partial_k v_j \qquad ((j, k, l)\text{: an even permutation of } (1, 2, 3)).$$

This can be shown in the same way as ZIMAN [13] did with respect to the transformed potentials Ψ^*, Ψ in his notation.

It seems that the deviation of density $\varrho' = \varrho - \varrho_0$ from its mean value ϱ_0 is small, so we regard ϱ' as the density variable and neglect the higher order terms with respect to ϱ'/ϱ_0. In particular we can know the form of $V(\varrho)$ only as a power series of ϱ'. Since the hydrodynamical field describes collective motions of a great assembly of innumerable particles, the ordinary treatments of quantization with respect to the density deviation seems to be reasonable physically. As ϱ_0 is a constant number ϱ' satisfies the same commutation relations as ϱ. The Hamiltonian density can be written with the canonical variables as in (4.2).

5. – The energy spectrum of hydrodynamical quanta.

Our task in this section is to diagonalize the Hamiltonian (4.2) and to obtain the energy spectrum of the quanta for the quantized hydrodynamical field. When we perform the functional and the coordinate Fourier transformations:

$$\Psi \to \int \widetilde{\Psi} \exp\left[i\hbar \int \widetilde{\xi}\widetilde{\varphi}\,\mathrm{d}k' + i\hbar \int \widetilde{\eta}^-\,\widetilde{\psi}\,\mathrm{d}k''\right] \mathrm{d}\widetilde{\varphi}\,\mathrm{d}\widetilde{\psi}\,, \tag{5.1}$$

the field operators transform as follows

$$\partial_l \varphi \sim \hbar k_l \widetilde{\xi}^*\,, \qquad \varrho \sim -\widetilde{\xi}\,, \qquad [\widetilde{\xi}^*(k), \widetilde{\xi}(k')] = \delta(k - k')\,, \tag{5.2a}$$

$$\partial_l \psi \sim \hbar k_l \widetilde{\eta}^+\,, \qquad \sigma \sim -\widetilde{\eta}^-\,, \qquad [\widetilde{\eta}^+(k), \widetilde{\eta}^-(k')] = \delta(k - k')\,, \tag{5.2b}$$

Next we perform a transformation for field operators $\widetilde{\eta}^+$, $\widetilde{\eta}^-$ to $\widetilde{\eta}^*$, $\widetilde{\eta}$

$$\widetilde{\eta}^+ = \tfrac{1}{2}\widetilde{\eta}^* + \widetilde{\eta}\,, \qquad \widetilde{\eta}^- = -\tfrac{1}{2}\widetilde{\eta}^* + \widetilde{\eta}\,, \tag{5.3}$$

which satisfy the commutation relation

$$[\widetilde{\eta}^*(k), \widetilde{\eta}(k')] = \delta(k - k')\,. \tag{5.4}$$

The problem concerning the dimension in the equation (5.3) is not serious, for we can easily justify all the important quadratic relations by using unit constants ε_+, ε_- of the same dimension as η_+, η_-. After these transformations the Hamiltonian density becomes

$$\widetilde{H} = \frac{\hbar^2 k^2}{2}\{\widetilde{\xi}\widetilde{\xi}^{*2} + 2\widetilde{\xi}^*(-\tfrac{1}{2}\widetilde{\eta}^* + \widetilde{\eta})(\tfrac{1}{2}\widetilde{\eta}^* + \widetilde{\eta}) + \\ + \widetilde{\xi}^{-1}((-\tfrac{1}{2}\widetilde{\eta}^* + \widetilde{\eta})(\tfrac{1}{2}\widetilde{\eta}^* + \widetilde{\eta}))^2\} + \widetilde{V}\,. \tag{5.5}$$

In the representation where $\tilde{\xi}$, $\tilde{\eta}$ are diagonal the functional Schrödinger equation becomes

$$(5.6)\qquad \left[-\hbar^2k^2\left\{\frac{1}{2}\tilde{\xi}\frac{\delta^2}{\delta\tilde{\xi}^2}+\frac{\delta}{\delta\tilde{\xi}}\left(-\frac{1}{2}\frac{\delta}{\delta\tilde{\eta}}+\tilde{\eta}\right)\left(\frac{1}{2}\frac{\delta}{\delta\tilde{\eta}}+\tilde{\eta}\right)+\right.\right.$$
$$\left.\left.+\frac{1}{2}\tilde{\xi}^{-1}\left(\left(-\frac{1}{2}\frac{\delta}{\delta\tilde{\eta}}+\tilde{\eta}\right)\left(\frac{1}{2}\frac{\delta}{\delta\tilde{\eta}}+\tilde{\eta}\right)\right)^2\right\}+\tilde{V}\right]\tilde{\Psi}(\tilde{\xi},\tilde{\eta})=\tilde{\varepsilon}\tilde{\Psi}\partial\tilde{\xi},\tilde{\eta}) .$$

If we put

$$(5.7)\qquad \tilde{\Psi}(\tilde{\xi},\tilde{\eta})=\tilde{\Phi}(\tilde{\xi})J'_m(\tilde{\eta}) ,$$

with the Hermite functional J'_m, satisfying

$$(5.8)\qquad \left(\frac{1}{2}\frac{\delta}{\delta\tilde{\eta}}+\tilde{\eta}\right)\left(\frac{1}{2}\frac{\delta}{\delta\tilde{\eta}}+\tilde{\eta}\right)J'_m\{\tilde{\eta}\}=mJ'_m\{\tilde{\eta}\} ,$$

we have the equation

$$(5.9)\qquad \left[-\hbar^2k^2\left\{\frac{1}{2}\tilde{\xi}\frac{\delta^2}{\delta\tilde{\xi}^2}+m\frac{\delta}{\delta\tilde{\xi}}+\frac{m^2}{2}\tilde{\xi}-1\right\}+\tilde{V}\right]\tilde{\Psi}(\tilde{\xi})=\tilde{\varepsilon}\tilde{\Psi}(\tilde{\xi}) .$$

If the equation of state for the fluid is known, we can determine $\tilde{V}(\tilde{\xi})$. By solving the functional equation (5.9) we can obtain the energy spectrum in the form $E(n, m, k)$, where n is the index indicating the eigenvalues concerning the equation (5.9). In the case of a liquid, the density will be nearly constant and expression (4.3) for V can give a good approximation. In this case we must use also the treatment of non-locality introduced in our previous report [34]. Now by a transformation

$$(5.10)\qquad \tilde{\xi}\to\tilde{\xi}'=\tilde{\xi}-\tilde{\xi}^0 ,$$

$$(5.11)\qquad \tilde{\Phi}(\tilde{\xi})\to\tilde{\Phi}'(\tilde{\xi}')=\tilde{\xi}^m\tilde{\Phi}(\tilde{\xi}) ,$$

we have in the approximation $(\tilde{\xi}'/\tilde{\xi}^0)^n \ll 1,\ n>2$

$$(5.12)\qquad \left\{\frac{\delta^2}{\delta\tilde{\xi}'^2}-\alpha\tilde{\xi}'-\beta\tilde{\xi}'+\gamma\right\}\tilde{\Phi}'(\tilde{\xi}')=0 ,$$

$$(5.13a)\qquad \alpha=\frac{1}{\tilde{\xi}^{0 3}}\left(\frac{1}{\hbar^2k^2}(\tilde{\xi}^0c_0^2-2\tilde{\varepsilon})-\frac{3m}{\tilde{\xi}^0}\right)$$

$$(5.13b)\qquad \beta=\frac{1}{2\tilde{\xi}_0^2}\left(\frac{\tilde{\varepsilon}}{\hbar^2k^2}+\frac{m}{\tilde{\xi}^0}\right)$$

$$(5.13c)\qquad \gamma=\frac{1}{\tilde{\xi}_0}\left(\frac{2\tilde{\varepsilon}}{\hbar^2k^2}+\frac{m}{\tilde{\xi}_0}\right)$$

(33) L. I. Schiff: *Phys. Rev.*, **84**, 1 (1951).
(34) K. Gotō: *Prog. Theor. Phys.*, **15**, No. 2 (1956).

By a further transformation

$$\tilde{\xi}'' = \alpha^{\frac{1}{4}}(\tilde{\xi}' + \beta/2\alpha) \tag{5.14}$$

we have

$$\left\{\frac{\delta^2}{\delta\tilde{\xi}''^2} - \tilde{\xi}''^2 + \frac{\beta^2 + 4\alpha\gamma}{4\alpha^{\frac{3}{2}}}\right\}\tilde{\Phi}'(\tilde{\xi}'') = 0\,. \tag{5.15}$$

Thus we have

$$\tilde{\Phi}'(\tilde{\xi}'') = J_n(\tilde{\xi}'')\,, \tag{5.16}$$

$$\frac{\beta^2 + 4\alpha\gamma}{4\alpha^{\frac{3}{2}}} = 2n + 1\,. \tag{5.17}$$

From the last relation we have the energy spectrum

$$\tilde{\varepsilon}(k) = -\beta' + \sqrt{\beta'^2 + \gamma'} \doteqdot \gamma'/2\beta'\,, \tag{5.18}$$

$$\beta' = \hbar^2 k^2 (16\tilde{\xi}_0^3 \alpha + m/\tilde{\xi}^0)\,, \tag{5.18a}$$

$$\gamma' = \hbar^4 k^4 (16\alpha^{\frac{3}{2}} \tilde{\xi}^{0^4}(2n+1) - 16 m\alpha\tilde{\xi}^{0^3} - m^2/\tilde{\xi}^{0^2})\,. \tag{5.18b}$$

In the case of $m = 0$

$$\left\{\begin{aligned} &\beta' = 16\tilde{\xi}^0 C_0^2\,,\\ &\gamma' = 16\alpha\hbar^4 k^4(\alpha^{\frac{1}{2}}\tilde{\xi}^{0^4}(2n+1) - \tilde{\xi}^{0^2})\,,\\ &\alpha = (\tilde{\xi}^0 C_0^2 - 2\tilde{\varepsilon})/\tilde{\xi}^{0^3}\hbar^2 k^2 \doteqdot C_0^2/\tilde{\xi}^{0^2}\hbar^2 k^2 \qquad (\text{for } \tilde{\varepsilon} \ll \tilde{\xi}^0 C_0^2) \end{aligned}\right. \tag{5.19}$$

So in this case for small k, by neglecting higher terms in power series expansions in k, we have

$$\varepsilon(k) = \hbar k C_0 (n + \tfrac{1}{2})\,. \tag{5.20}$$

This is the same as the so-called phonon spectrum. Moreover even for $m \neq 0$ the terms involving m are small for small values of k and in this case also we have the same result as (5.20). Thus we see that for small values of k the spectrum of the hydrodynamical quantum is the same as those of the phonon in any case.

When k is not so small, we must solve the equation (5.18) numerically. This is somewhat tedious, for α and so γ' involve $\tilde{\varepsilon}$. Although the spectrum for very small k was independent of $\tilde{\xi}_0$, it is involved in the spectrum in general. Here we do not use the method of cut off, because the method of Debye using the degrees of freedom for motions of particles, which was meaningful for

small linear vibrations or free motions, seems to be doubtful for the hydrodynamical case. We will introduce the non locality in the way discussed in our previous report [34]. If we assume the non locality of the extension of volume for one He atom, we can determine $\tilde{\xi}^0$ using the value of ϱ_0 for He in the state concerned. The obtained curve for the spectrum has a tendency almost similar to that expected by LANDAU, even if it has not a so sharp minimum point as that. The last circumstance seems to be an unsufficient point of the purely hydrodynamical picture of the phonon and the roton. For the interpretation of the energy spectrum of He II some atomistic considerations seem to be necessary.

III. Non Linear Meson Theory.

6. – The non linear meson field of Schiff's type.

A trial of the non-linear meson theory was given by SCHIFF [33] for the first time in order to explain the properties of nuclear forces particularly of the saturation. The formulation used there was a non relativistic and classical theory. The quantization of such a field was performed subsequently by him with the method of the lattice space quantization [5].

The Hamiltonian density of the non-linear meson field of Schiff's type is

$$H = \tfrac{1}{2}[\pi^2 + (\nabla\varphi)^2 + \varkappa^2\varphi^2 + \tfrac{1}{2}\alpha\varphi^4]\,, \tag{6.1}$$

where φ is a real scalar field of the meson with the mass $\varkappa$, π is its conjugate momentum and α is the non-linear weight function ($\hbar = c = 1$). In our formulation the quantization is performed according to the general procedure of (2.3)

$$\begin{cases} [\pi(\bar{x}, x_4), \varphi(\bar{x}', x_4)] \quad = i\delta(\bar{x} - \bar{x}') \\ [\pi(\bar{x}, x_4), \partial_l\varphi(\bar{x}', x_4)] = -\delta^{(l)}(\bar{x} - \bar{x}') \\ [\varphi(\bar{x}, x_4), \partial_l\varphi(\bar{x}', x_4)] = 0 \end{cases} \tag{6.2}$$

and the functional Schrödinger equation is established, that is in the representation where π is diagonal

$$\frac{1}{2}\left[\pi^2 - \left(\frac{\delta^{(l)}}{\delta^{(l)}\pi}\right)^2 - \varkappa^2\left(\frac{\delta}{\delta\pi}\right)^2 + \frac{1}{2}\alpha\left(\frac{\delta}{\delta\pi}\right)^4\right]\Psi(\pi(x)) = \varepsilon(x)\Psi(\pi(x))\,. \tag{6.3}$$

To solve this functional equation is not so easy because of the new differential operator $\delta^{(l)}/\delta^{(l)}\pi$, and so we must perform the generalized Fourier transformations

$$\Psi(\pi(x)) \to \int\tilde{\Psi}(\tilde{\chi}(k)) \exp\left[-i\int\tilde{\chi}(k')\tilde{\pi}(k')\right]\mathrm{d}k'\,\mathrm{d}\pi(k)\,. \tag{6.4}$$

After this transformation the Schrödinger equation becomes for even or odd functions

$$\frac{1}{2}\left[-\frac{\delta^2}{\delta\tilde{\chi}^2}+(k^2+\varkappa^2)\tilde{\chi}^2+\frac{1}{2}\tilde{\alpha}\tilde{\chi}^4\right]\tilde{\Psi}(\tilde{\chi})=\tilde{\varepsilon}\tilde{\Psi}(\tilde{\chi})\,. \tag{6.5}$$

The boundary condition is the square integrability of the state functional $\tilde{\Psi}$. This functional equation is linear and seems to have the eigenvalue problem of the Sturm-Liouvilles type according to its form. If $\alpha=0$ we can easily solve it with the Hermite functional. However in the general case ($\alpha\neq 0$) it is not easy to obtain the exact solution and we must use methods of approximation. Such a method may be that of series expansions or of W-K-B. Anyhow the solution will be given by the functional composed by referring to the eigenfunction of the corresponding usual equation of one variable [24]

$$\frac{1}{2}\left[-\frac{d^2}{dy^2}+(k^2+\varkappa^2)y^2+\frac{1}{2}y\tilde{\alpha}^4\right]\psi(y)=\tilde{\varepsilon}\psi(y)\,, \tag{6.6}$$

with the boundary condition of the square integrability of $\psi(y)$. If $\psi(y)$ has a series expansion in terms of Hermite functions, which form a complete orthogonal set for the square integrable functions, then the solving functional $\tilde{\Psi}(\tilde{\chi})$ will be obtained by the corresponding series in terms of the Hermite functionals. Thus the energy spectrum is similar to that of (6.6). Since the equation (6.6) is of the same type as the wave equation for the anharmonic oscillator, we can say, at least concerning the energy spectrum, that the quantized non linear meson field of Schiff's type is composed of anharmonic oscillators in the meaning of the interpretation of the quantized electromagnetic field as an assembly of harmonic oscillators.

The equation of (6.6) is essentially the same as that investigated by SCHIFF, and this will give a field theoretical basis to it. In our treatment all the troubles concerning the lattice such as the discontinuity or the periodicity do not occur. For example the momentum operator can be defined with the ordinary form

$$G_l=-\tfrac{1}{2}[\pi(\nabla\varphi)+(\nabla\varphi)\pi]$$

and in our χ representation it is

$$\tilde{G}_l=-\frac{k_l}{2}\left[\frac{\delta}{\delta\tilde{\chi}}\tilde{\chi}+\tilde{\chi}\frac{\delta}{\delta\tilde{\chi}}\right]. \tag{6.7}$$

With this orthodox form the troubles, which occurred in the discontinuous cases of SCHIFF [5] and HOLLAND [6], do not arise in our continuous case.

APPENDIX

Products in the trasformed representation.

In the following we will show how the multiplication law, for some class of Hamiltonian functions, in the new representation introduced after the coordinate—and functional—Fourier transformations may restore the original form.

(i) *Free electromagnetic field.* – After the transformation

$$E_l(x) = \int \tilde{E}_l(k) \exp[ikx]\, \mathrm{d}k \tag{A.1}$$

$$\Psi(E(x)) = \int \tilde{\Psi}(\tilde{E}_l(k)) \exp\left[i \int \tilde{\chi}_l(k')\, \tilde{E}_l(k')\, \mathrm{d}k'\right] \mathrm{d}\tilde{E}(k) \exp[-2ikx]\, \mathrm{d}k \tag{A.2}$$

the equation (3.7) becomes, for even or odd $\tilde{E}_l(k)$,

$$\begin{aligned}
&\int \frac{1}{2}\{E_l(x) E_l(x) - \hbar^2 \partial_m A_l(x) \cdot \partial_m A_l(x)\} \tilde{\Psi}(\tilde{\chi}_l(k)) \exp\Big[i \cdot \\
&\quad \cdot \int \tilde{\chi}_l(k')\, \tilde{E}_l(k')\, \mathrm{d}k'\Big] \Pi\, \mathrm{d}\tilde{E}_l(k) \exp[-2ikx]\, \mathrm{d}k\, \mathrm{d}x = \\
&= \int \frac{1}{2}\left\{\tilde{E}_l(k'')\tilde{E}_l(k''') - \hbar^2 k''_m k'''_m \frac{\delta}{\delta \tilde{E}_l(k'')} \frac{\delta}{\delta \tilde{E}_l(k''')}\right\} \tilde{\Psi}(\tilde{\chi}_l(k)) \cdot \\
&\quad \cdot \exp\left[i \int \tilde{\chi}(k')\, \tilde{E}(k')\, \mathrm{d}k'\right] \mathrm{d}\tilde{E}(k)\, \delta(k'' + k''' - 2k)\, \mathrm{d}k''\, \mathrm{d}k'''\, \mathrm{d}k = \\
&= \int \frac{1}{2}\left\{- \frac{\delta}{\delta \tilde{\chi}_l(k'')} \frac{\delta}{\delta \tilde{\chi}_l(k''')} - \hbar^2 k''_m k'''_m \tilde{\chi}_l(k'') \tilde{\chi}_l(k''')\right\} \delta(k'' - k')\, \delta(k''' - k') \cdot \\
&\quad \cdot\, \delta(k'' + k''' - 2k) \tilde{\Psi}(\tilde{\chi}_l(k)) \exp\left[i \int \tilde{\chi}(k')\, \tilde{E}(k')\, \mathrm{d}k'\right] \mathrm{d}\tilde{\chi}(k)\, \mathrm{d}k'\, \mathrm{d}k''\, \mathrm{d}k'''\, \mathrm{d}k = \\
&= \int \frac{1}{2}\left\{- \frac{\delta}{\delta \tilde{\chi}_l(k)} \frac{\delta}{\delta \tilde{\chi}_l(k)} + \hbar^2 k^2 \tilde{\chi}_l(k) \tilde{\chi}_l(k)\right\} \cdot \\
&\quad \cdot \tilde{\Psi}(\tilde{\chi}_l(k)) \exp\left[i \int \tilde{\chi}(k')\, \tilde{E}(k')\, \mathrm{d}k'\right] \mathrm{d}\tilde{\chi}(k)\, \mathrm{d}k\, .
\end{aligned} \tag{A.3}$$

This is the left hand side of the equation (3.9).

(ii) *Hydrodynamical field.* – After the transformation

$$
\text{(A.4)}\quad \begin{cases}
\varrho(x) = \int \tilde{\varrho}(k) \exp[ikx]\,\mathrm{d}k\,, \\
\varphi(x) = \int \tilde{\varphi}(k) \exp[-ikx]\,\mathrm{d}k\,, \\
\sigma(x) = \int \tilde{\sigma}(k) \exp[ikx]\,\mathrm{d}k\,, \\
\psi(x) = \int \tilde{\psi}(k) \exp[-ikx]\,\mathrm{d}k\,, \\
V(x) = \int \tilde{V}(k) \exp[-ikx]\,\mathrm{d}k\,, \\
\Psi(\varphi(x), \psi(x)) = \int \tilde{\Psi}(\tilde{\varphi}(k), \tilde{\psi}(k)) \cdot \\
\quad \cdot \exp\left[i\hbar \int \tilde{\xi}(k')\tilde{\varphi}(k')\,\mathrm{d}k' + i\hbar \int \tilde{\eta}^-(k'')\,\tilde{\psi}(k'')\,\mathrm{d}k''\right] \mathrm{d}\tilde{\varphi}(k)\,\mathrm{d}\Psi(k) \exp[ikx]\,\mathrm{d}k
\end{cases}
$$

we have

$$
\begin{aligned}
\text{(A.5)}\quad \int H(x)\Psi(x)\,\mathrm{d}x = \int \frac{1}{2}\,[\{\tilde{\varrho}(k^{(1)})k_m^{(2)}k_m^{(3)}\tilde{\varphi}(k^{(2)})\tilde{\varphi}(k^{(3)})\,\delta k^{(1)} - k^{(2)} - k^{(3)} + k) + \\
+ k_m^{(1)}k_m^{(3)}\tilde{\varphi}(k^{(1)})\tilde{\sigma}(k^{(2)})\tilde{\psi}(k^{(3)})\,\delta(-k^{(1)} + k^{(2)} - k^{(3)} + k)\}\,\mathrm{d}k^{(1)}\,\mathrm{d}k^{(2)}\,\mathrm{d}k^{(3)} + \\
+ \{\tilde{\varrho}^{-1}(k^{(1)})\tilde{\sigma}(k^{(2)})k_m^{(4)}k_m^{(5)}\tilde{\psi}(k^{(4)})\tilde{\sigma}(k^{(3)})\tilde{\psi}(k^{(5)})\,\delta(-k^{(1)} + k^{(2)} - k^{(4)} + k^{(3)} - k^{(5)} + k)\} \cdot \\
\cdot\, \mathrm{d}k^{(1)}\mathrm{d}k^{(2)}\mathrm{d}k^{(3)}\mathrm{d}k^{(4)}\mathrm{d}k^{(5)} + \tilde{V}(k)]\,\tilde{\Psi}(\tilde{\xi}(k), \tilde{\eta}^-(k)) \cdot \\
\cdot \exp\left[i\hbar \int \tilde{\xi}(k')\tilde{\varphi}(k')\,\mathrm{d}k' + i\hbar \int \tilde{\eta}^-(k'')\,\tilde{\psi}(k'')\,\mathrm{d}k''\right] \mathrm{d}\tilde{\varphi}(k)\,\mathrm{d}\tilde{\psi}(k)\,\mathrm{d}k = \\
= \int \frac{1}{2}\,k^2[\tilde{\xi}(k)\tilde{\xi}^*(k)^2 + 2\tilde{\xi}^*(k)\tilde{\eta}^-(k)\tilde{\eta}^+(k) + \tilde{\xi}^{-1}(\tilde{\eta}^-(k)\tilde{\eta}^+(k))^2 + \tilde{V}(k)] \cdot \\
\cdot \tilde{\Psi}(\tilde{\xi}(k), \tilde{\eta}^-(k)) \exp\left[i\hbar \int \tilde{\xi}(k')\,\tilde{\varphi}(k')\,\mathrm{d}k' + i\hbar \int \tilde{\eta}^-(k'')\tilde{\psi}(k'')\,\mathrm{d}k''\right] \mathrm{d}\tilde{\varphi}(k)\,\mathrm{d}\tilde{\psi}(k)\,\mathrm{d}k.
\end{aligned}
$$

This is the relation leading to the equation (5.5).

(iii) *Non-linear meson field.* – The situation is quite similar to the case (i). The only care must be taken for $\tilde{\alpha}$:

$$
\text{(A.6)}\qquad \alpha(x) = \int \tilde{\alpha}(k) \exp[-2ikx]\,\mathrm{d}k\,.
$$

The product $\alpha\varphi^4$ is not an ordinary one but has a special form $\alpha\cdot\varphi^4$, where

$$
A\cdot B(x) = \int A(x - 2x')B(x')\,\mathrm{d}x'.
$$

This is not so extraordinary, because the energy concerned becomes

$$\int \alpha \cdot \varphi^4(x)\,\mathrm{d}x = \int \alpha(x - 2x')\varphi^4(x')\,\mathrm{d}x\,\mathrm{d}x'.$$

By this product the separation in the k space becomes possible.

RIASSUNTO (*)

Si propone un metodo per lo sviluppo in rappresentazione funzionale delle teorie dei campi non lineari quantizzati, quando la non linearità dell'equazione classica appare in una forma particolare degli operatori di campo, e l'equazione del campo pei vettori di stato diventa un'equazione differenziale lineare dei funzionali. Il metodo di diagonalizzazione dell'hamiltoniana del campo è illustrato per mezzo del cmpo elettromagnetico libero, de. campo idrodinamico e del campo mesonico non lineare.

(*) *Traduzione a cura della Redazione.*

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A Relativistic Theory of Charged Particles in an Electromagnetic and Gravitational Field.

H. T. Flint

Bedford College, University of London

E. M. Williamson

St. Mary's College, University of Durham

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Summary. — A theory of charged particles is developed by an extension of Einstein's concept of local axes. A new four vector is added to the geometrical system in addition to the electromagnetic potential vector. The particle is then identified with the geometry and the discussion leads to a relativistic theory of the electron, in which the difficulty concerning the self-energy is avoided.

Theories of the electromagnetic field and of the electrically charged particle are of two kinds. On the one hand there are those that follow H. A. Lorentz in regarding the charged particle as the generator of the field in which the field intensities and potential are expressed in terms of the charge and motion of the particle and on the other those which, like the theory of G. Mie, regard the field quantities as primary, the charge being an expression of the field.

In this paper the theory is based on the second of these views. It follows the relativity theory of gravitation in its use of geometrical concepts and in its appeal to the forms of Riemannian geometry. In the theory of relativity use is made of the test particle, that is to say, the particle, whose motion is considered, is regarded as in no way disturbing the geometry of the continuum. Even the planets in the field of the sun are considered too small to modify it. In the present theory this is not the case; the particle modifies the geometry even if it moves in an external field in comparison with which its own field may

be neglected. At first the case of the external field will be considered and later the case of the field of the particle itself.

The object is to identify certain quantities, such as momentum, with the field, as MIE attempts to do in his theory, but the identification is now with the geometry, which is the expression both of the field and the particle.

It does not appear to be possible to develop these ideas within the framework of a four-dimenisonal continuum such as that of the theory of general relativity. It is necessary to make use of a geometry of higher dimensions and the present purpose can be served by means of a five-dimensional continuum or a system of projective geometry. If the limitations of KALUZA (1) and KLEIN (2) be adopted, it is possible to express all the results in the familiar covariant forms of four dimensions and the same is true, if the projective geometry of B. HOFFMANN (3) and O. VEBLEN be used. The Kaluza and Klein theory will be applied here and a brief account of this theory with a suggested modification will be given first. This will be followed by a new development, based on the notation of Einstein's theory of distant parallelism (4), which is an essential part of the theory.

1. – The theory of Kaluza and Klein.

The fact that electromagnetic phenomena, unlike those of gravitation, could not be recognized as an expression of the geometry of space was recognised early as a difficulty in the theory of relativity. The equations of the electromagnetic field can be readily written in the covariant form but they are not incorporated in the general theory. One of the first attempts to remove this difficulty was that of KALUZA who showed that it was possible by means of a five-dimensional continuum to identify the path of a charged particle in a gravitational and electromagnetic field with the geodesic of the space. Both fields were in this case able to be regarded as expressions of the geometry. O. KLEIN (2) later showed that the notation introduced by KALUZA had applications in the quantum theory.

In order to carry out the purpose of the theory it is only necessary to make use of a limited group of transformations. These are:

$$x'^m = f_m(x^n)\,, \qquad x'^5 = x^5 + g(x^n)\,. \tag{1}$$

The Latin affixes m, n may have the values 1, 2, 3, 4; if a Greek letter is used

(1) TH. KALUZA: *Sitzgsber. preuss. Akad. Wiss.*, 966 (1921).
(2) O. KLEIN: *Zeits. f. Phys.*, **46**, 188 (1927).
(3) B. HOFFMANN and O. VEBLEN: *Phys. Rev.*, **36**, 810 (1930).
(4) A. EINSTEIN: *Sitzgsber. preuss. Akad. Wiss.*, **17**, 217 (1928).

as affix, e.g. μ, it may have the values 1, 2, 3, 4, 5. $f(x^n)$ means that the function depends on the four coordinates. With this group of transformations it is easy to relate four- and five-dimensional quantities [5], since the four components (A^m) of a five-vector (A^μ) transform like a four-vector. In order to distinguish between the number of dimensions, capital letters will be used for five-vectors and small letters for four-vectors. Thus

$$A^m = a^m . \tag{2}$$

Similarly the fifth component of the covariant (A_μ), viz. A_5, transforms like a four-dimensional scalar. The metric of the continuum is defined by

$$d\sigma^2 = \gamma_{\mu\nu} dx^\mu dx^\nu , \tag{3}$$

where $d\sigma$ is the line element.

This corresponds to the four-dimensional expression

$$ds^2 = g_{mn} dx^m dx^n .$$

The relation between the coefficients $(\gamma_{\mu\nu})$ and $g_{mn})$ which serve the purpose of Kaluza's theory are:

$$\left\{\begin{array}{lll} \gamma_{mn} = g_{mn} + \gamma_{55}\alpha^2\varphi_m\varphi_n , & \gamma_{m5} = \gamma_{55}\alpha\varphi_m , & \\ \gamma^{mn} = g^{mn}, & \gamma^{m5} = -\alpha\varphi^m, & \gamma^{55} = \dfrac{1}{\gamma_{55}} + \alpha^2\varphi_m\varphi^m, \end{array}\right. \tag{4}$$

where the (φ_m) are components of the electromagnetic potential. None of the quantities in the relations (4) depends upon the fifth coordinate.

In these relations α is a constant and different values have been assigned to it in the past. Here it will be assumed to have the value e/m_0c^2, where e is the fundamental unit of electric charge and m_0 the rest mass of the electron. Reasons suggesting this particular choice occur in the literature where it has usually been assumed that e denotes the charge of the particular particle under consideration and m_0 its rest mass. The assumption that α is a universal constant is a feature of the present theory which leads to some interesting consequences.

The way in which the fifth coordinate occurs in the quantities introduced into this theory is limited. It often does not occur, as in the expressions (4) but any quantity which depends on x^5 does so through the factor $\exp[2\pi i n' x^5/l_0]$, where n' is an integer, positive or negative and l_0 is the length

(5) H. T. FLINT: *Phil. Mag.*, 7, 36, 417 (1940).

h/m_0c, n' is to be identified with the charge on a particle present at a point of the continuum.

The relations (4) make it possible to relate four-and five-dimensional quantities.

Thus

$$(5)\qquad \begin{cases} A_m = a_m + \alpha\varphi_m A_5\,, \\ A^5 = \dfrac{A_5}{\gamma_{55}} + \alpha\varphi_m a^m\,. \end{cases}$$

The square of the line element can also be written in the form

$$(6)\qquad d\sigma^2 = ds^2 + (\gamma_{5\mu}dx^\mu)^2/\gamma_{55} = ds^2 + dx_5^2/\gamma_{55}\,.$$

Thus $d\sigma$ is represented as if it consisted of two components ds and $dx_5/\sqrt{\gamma_{55}}$ at right angles to one another, so that the fifth coordinate axes can be regarded as being normal to the four-dimensional continuum. It has been seen that dx_5 is a scalar from the point of view of four-dimensions and, corresponding to dx_5 the length $dx_5/\sqrt{\gamma_{55}}$ will be regarded as a four-dimensional scalar quantity. In the same way the four-dimensional quantity which corresponds to the vector component A_5 will be denoted by $a.$, with $a. = A_5/\sqrt{\gamma_{55}}$.

In the identification of the path of a charged particle with the geodesic of the continuum it is necessary to place

$$(6)\qquad \alpha\,\frac{dx_5}{d\tau} = \frac{q}{M_0c}\,,$$

where q is the charge of the particle and M_0 its rest mass. Place $q = n'e$ and $M_0 = nm_0$, where n' is a positive or negative integer and n denotes the ratio of the rest mass of the particle to that of the electron. The assumption that a is a universal constant equal to e/m_0c^2 leads to

$$(7)\qquad dx_5 = n'c\,d\tau/n\,,$$

which may be regarded as a relation which gives the physical significance of the fifth coordinate.

It has been suggested that the path of the particle should be regarded as a null-geodesic so that particles in five-dimensions are analogous to photons in four-dimensions [6]. If this suggestion be adopted it follows from (6) that

$$(8)\qquad \gamma_{55} = (n'/n)^2\,.$$

([6]) J. W. FISHER: *Proc. Roy. Soc.*, A **123**, 489 (1929).

This relation expresses the influence of the particle on the geometry of the continuum. It may be regarded as an addition to Kaluza's relations (4) and it represents the way in which the particle, by its charge and mass, modifies the local geometry. The momentum of the particle is given by

$$\pi_\mu = M_0 \gamma_{\mu\nu} \frac{\mathrm{d}x^\nu}{\mathrm{d}\tau},$$

where M_0 is the rest mass, and the fifth component is

$$\pi_5 = M_0 \gamma_{5\nu} \frac{\mathrm{d}x^\nu}{\mathrm{d}\tau} = n m_0 \frac{\mathrm{d}x_5}{\mathrm{d}\tau} = n' m_0 c\,, \tag{9}$$

by equation (7).

The corresponding four-dimensional quantity is

$$p. = \Pi_5/\sqrt{\gamma_{55}} = n m_0 c\,, \tag{10}$$

thus Π_5 is proportional to the charge and $p.$ to the mass of the particle.

The value of Π_5 agrees with the definition of momentum according to the quantum theory, for according to this the fifth component is $(h/2\pi i)\partial\psi/\partial x^5$, where ψ is the wave function. But since x^5 occurs in the form $\exp[2\pi i n' x^5/l_0]$,

$$\Pi_5 = hn'/l_0 = n' m_0 c\,.$$

2. – Local Axes [4,7].

In his theory of distant parallelism EINSTEIN introduced the idea that at any point in space it is possible to erect a set of orthogonal axes as a system of local reference. At the same time it is assumed to be possible to refer to a system of curvilinear axes. The former will be denoted by $(\boldsymbol{\epsilon}_\alpha)$, the latter by $(\boldsymbol{i}^\mu)$ or $(\boldsymbol{i}_\mu)$. For the present purpose it is necessary that α and μ should take the values 1 to 5.

A linear relation is assumed to exist between the $(\boldsymbol{i}^\mu)$ and $\boldsymbol{\epsilon}_\alpha)$, as follows:

$$\boldsymbol{i}^\mu = h^\mu{}_\alpha \boldsymbol{\epsilon}_\alpha\,, \qquad \boldsymbol{i}_\mu = h_{\mu\alpha} \boldsymbol{\epsilon}_\alpha\,. \tag{11}$$

With α fixed $h^\mu{}_\alpha$ and $h_{\mu\alpha}$ are components of a five vector. These coefficients will be supposed to be functions of the co-ordinates, including x^5. It is ne-

(7) H. T. FLINT: *Proc. Roy. Soc.*, A **121**, 676 (1928); V. BARGMANN: *Sitzgsber. preuss. Akad. Wiss.*, **24**, 346 (1932); H. W. HASKEY: *Phil. Mag.*, **7**, 30, 478 (1940).

cessary to consider the coefficients as complex quantities and to introduce the vectors

$$\bar{\boldsymbol{i}}^{\mu}=\bar{h}^{\mu}{}_{\alpha}\boldsymbol{\epsilon}_{\alpha} \quad \text{and} \quad \bar{\boldsymbol{i}}_{\mu}=\bar{h}_{\mu\alpha}\boldsymbol{\epsilon}_{\alpha}\,, \tag{12}$$

where $\bar{h}^{\mu}{}_{\alpha}$ denotes the complex conjugate of $h^{\mu}{}_{\alpha}$. The orthogonal unit vectors satisfy the relations:

$$\boldsymbol{\epsilon}_{\alpha}\cdot\boldsymbol{\epsilon}_{\beta}=\delta_{\alpha\beta}\,, \tag{13}$$

and the curvilinear vectors satisfy:

$$\begin{cases} \boldsymbol{i}^{\mu}\cdot\bar{\boldsymbol{i}}^{\nu}=\bar{\boldsymbol{i}}^{\mu}\cdot\boldsymbol{i}^{\nu}=\gamma^{\mu\nu}\,, \\ \bar{\boldsymbol{i}}^{\mu}\cdot\boldsymbol{i}_{\nu}=\boldsymbol{i}^{\mu}\cdot\bar{\boldsymbol{i}}_{\nu}=\delta^{\mu}{}_{\nu}\,, \\ \boldsymbol{i}_{\mu}\cdot\bar{\boldsymbol{i}}_{\nu}=\bar{\boldsymbol{i}}_{\mu}\cdot\boldsymbol{i}_{\nu}=\gamma_{\mu\nu}\,. \end{cases} \tag{14}$$

Scalar products of the type $\boldsymbol{i}^{\mu}\cdot\boldsymbol{i}^{\nu}$ are not contemplated in the notation.

From these relations it follows that:

$$\begin{cases} \bar{h}^{\mu}{}_{\alpha}h_{\nu\alpha}=h^{\mu}{}_{\alpha}\bar{h}_{\nu\alpha}=\delta^{\mu}{}_{\nu}\,, & \bar{h}^{\mu}{}_{\alpha}h^{\nu}{}_{\alpha}=h^{\mu}{}_{\alpha}\bar{h}^{\nu}{}_{\alpha}=\gamma^{\mu\nu}\,, \\ \bar{h}_{\mu\alpha}h_{\nu\alpha}=h_{\mu\alpha}\bar{h}_{\nu\alpha}=\gamma_{\mu\nu}\,, & \bar{h}^{\mu}{}_{\alpha}h_{\mu\beta}=h^{\mu}{}_{\alpha}\bar{h}_{\mu\beta}=\delta_{\alpha\beta}\,. \end{cases} \tag{15}$$

The quantities on the right hand side of these relations are real so that the coefficients $(h^{\mu}{}_{\alpha})$ must be restricted to a particular form. The equations can be satisfied by writing

$$h^{\mu}{}_{\alpha}=f^{\mu}{}_{\alpha}\exp\left[\frac{2\pi i}{h}\right]\int\mathrm{d}\lambda\,, \tag{16}$$

where

$$\mathrm{d}\lambda=\frac{e}{c}\,\Theta_{\mu}\mathrm{d}x^{\mu}\,. \tag{17}$$

$f^{\mu}{}_{\alpha}$ and Θ_{μ} are real functions of the four coordinates $(x'\dots x^4)$ but $\mathrm{d}\lambda$ is not regarded as a perfect differential. It is thus necessary to prescribe a particular path of integration in order to obtain a definite value of $h^{\mu}{}_{\alpha}$. Such a path is assumed to be of physical importance. The indeterminate character of the $(h^{\mu}{}_{\lambda})$ has no influence upon the coefficients $(\gamma_{\mu\nu})$, so that it does not affect the description of gravitational phenomena, but it appears in those cases where the coefficients $(h^{\mu}{}_{\alpha})$ are individually concerned in the description.

The introduction of the factor e/c in the expression for $\mathrm{d}\lambda$ is suggested by

the term $e\varphi_m/c$ in many equations of the quantum theory and the vector (Θ_μ) is a generalization of the electromagnetic potential vector (φ_m).

The introduction of Planck's constant is then suggested by the fact that $e\Theta/hc$ is of the dimensions of the reciprocal of a length as is required of a quantity which is a factor of $\mathrm{d}x^\mu$ in the index of the exponential.

If ϱ denote the ratio $h^\mu{}_\alpha/f^\mu{}_\alpha$, it follows that

$$\frac{\mathrm{d}\varrho}{\varrho} = \frac{2\pi i e}{hc}\,\Theta_\mu\,\mathrm{d}x^\mu\,, \tag{18}$$

which recalls the relation

$$\frac{\mathrm{d}l}{l} = \varphi_m\,\mathrm{d}x^m\,,$$

expressing the change of length of a vector on a parallel displacement in Weyl's theory of gauging proposed for the purpose of including electromagnetic phenomena into the theory of relativity. The coefficients $h^\mu{}_\lambda$ etc., occurring in the formulae (15) may now be replaced by $f^\mu{}_\alpha$ etc.

Thus

$$f^\mu{}_\alpha f_{\nu\alpha} = \delta^\mu{}_\nu\,, \qquad f^\mu{}_\alpha f^\nu{}_\alpha = \gamma^{\mu\nu}\,.$$

From the above relations it follows that

$$\boldsymbol{\epsilon}_\alpha = \bar{h}_{\mu\alpha}\boldsymbol{i}^\mu = \bar{h}^\mu{}_\alpha\boldsymbol{i}_\mu = h_{\mu\alpha}\bar{\boldsymbol{i}}^\mu = h^\mu{}_\alpha\bar{\boldsymbol{i}}_\mu\,. \tag{19}$$

A vector $\boldsymbol{A}$ can be expressed in two ways:

$$\boldsymbol{A} = A_\mu\boldsymbol{i}^\mu = A_\alpha\boldsymbol{\epsilon}_\alpha\,.$$

It follows that

$$A_\alpha = h^\mu{}_\alpha A_\mu = h_{\mu\alpha}A^\mu\,, \quad A_\mu = \bar{h}_{\mu\alpha}A_\alpha\,, \quad A^\mu = \bar{h}^\mu{}_\alpha A_\alpha\,. \tag{20}$$

Similar results may be obtained by expressing $\boldsymbol{A}$ in the form

$$\boldsymbol{A} = \bar{A}_\mu\bar{\boldsymbol{i}}^\mu = A_\alpha\boldsymbol{\epsilon}_\alpha\,.$$

A similar notation can be taken over for relations between certain matrices. Let (γ^μ) be a set of five matrices corresponding to the vectors $(\boldsymbol{i}^\mu)$ dependent upon the coordinates and satisfying the Tetrode relations:

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\gamma^{\mu\nu}\,. \tag{21}$$

If $\gamma_\mu = \gamma_{\mu\nu}\gamma^\nu$, it follows that

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\gamma_{\mu\nu} .$$

Let (E_α) denote a set of five matrices with constant components corresponding to the orthogonal vectors $(\boldsymbol{\epsilon}_\alpha)$ and write

$$\gamma^\mu = h^\mu{}_\alpha E_\alpha , \qquad E_\alpha = \bar{h}_{\mu\alpha}\gamma^\mu , \quad \text{etc.} \tag{22}$$

If

$$E_\alpha E_\beta + E_\beta E_\alpha = 2\delta_{\alpha\beta} ,$$

the coefficients satisfy the set of values given by the relations (15). In order to make it possible to express results by the use of this analysis in terms of four-dimensional covariant relations some additional restrictions will be made. These are made in agreement with the limitations adopted by KALUZA in his choice of coefficients $(\gamma_{\mu\nu})$. It is assumed that the fifth axis of coordinates $\boldsymbol{\epsilon}_5$ is perpendicular to four-dimensional space and this is interpreted in the sense that the vectors $(\boldsymbol{i}^m)$, with $m = 1, 2, 3, 4$, have no components along $\boldsymbol{\epsilon}_5$.

Thus

$$h^m{}_5 = 0 .$$

Similarly the axis $\boldsymbol{i}_5$ will be supposed to have no components along the axes $\boldsymbol{\epsilon}_a$, with $a = 1, 2, 3, 4$.

Thus

$$h_{5a} = 0 .$$

Since $h_{5\alpha}\bar{h}_{5\alpha} = h_{55}\bar{h}_{55} = \gamma_{55}$, it follows that $f_{55} = \sqrt{\gamma_{55}}$.

From $h_{m\alpha}\bar{h}_{5\alpha} = \gamma_{m5} = \gamma_{55}\alpha\varphi_m$, it follows that $f_{m5} = \sqrt{\gamma_{55}}\alpha\varphi_m$.

Other results can be obtained in the same way so that the following table of values may be derived:

$$\begin{cases} f^m{}_5 = 0 , \quad f_{5a} = 0 , \quad f_{55} = \sqrt{\gamma_{55}} , \quad f^5{}_5 = 1/\sqrt{\gamma_{55}} , \\ f_{m5} = \sqrt{\gamma_{55}}\alpha\varphi_m , \quad f^5{}_a = -\alpha f_{ma}\varphi^m = -\alpha f^m{}_a\varphi_m . \end{cases} \tag{23}$$

These values give to some coefficients properties which they would not in general possess. The four-dimensional counterpart of f_{m5} is $(f_{m5} - \alpha\varphi_m f_{55})$ and is thus zero. The four-dimensional counterpart of f_{ma} is $(f_{ma} - \alpha\varphi_m f_{5a})$, so that since $f_{5a} = 0$, f_{ma} is its own four-dimensional counterpart.

It thus follows that

$$f^m{}_a f^n{}_a = g^{mn} , \qquad f^m{}_a f_{na} = \delta^m{}_n , \qquad f_{ma} f_{na} = g_{mn} . \tag{24}$$

3. – Application to a Relation from the Quantum Theory.

It has been suggested that the function ψ of Dirac's equation is to be regarded as a gauging factor [8]. A matrix of the form $A_\mu\gamma^\mu$ has the magnitude $\psi^+A_\mu\gamma^\mu\psi$, (A_μ) being a vector, and the change in this quantity when (A_μ) undergoes a parallel displacement is examined. It can be shown that, if it is zero, ψ satisfies Dirac's equation [9]. This procedure is an application of Weyl's concept, which was mentioned above, for the inclusion of electromagnetic phenomena in the theory of relativity by means of a system of gauging. The result of this consideration gives the same significance to the wave function as that which results from Born's suggestion but while the latter has been regarded as emphasizing the statistical aspect of quantum phenomena, the former lays emphasis on the fact that these phenomena express a particular system of measurement.

In the course of the development of this idea the matrix

$$K_{\mu\nu} = \frac{\partial\gamma_\mu}{\partial x^\nu} - \Delta^\varrho_{\mu\nu}\,\gamma_\varrho \tag{25}$$

appears, where $\Delta^\varrho_{\mu\nu}$ is the Christoffel bracket expression of the five-dimensional continuum. By analogy with the Riemann-Christoffel tensor, it is expected that, in expressing a law of the metric, some limitation is to be imposed upon it. As in the case of gravitation EINSTEIN expresses the law by $G_{\mu\nu} = 0$, so here the law is assumed to be

$$K^\mu{}_\mu = 0\,. \tag{26}$$

This leads immediately to Dirac's equation, and the condition (26) can be regarded as a fundamental law of the quantum theory. By means of the relations (22) and after expressing $\Delta^\varrho_{\mu\nu}$ in terms of the $(h^\mu{}_\lambda)$ by means of (15), it follows that

$$K^\mu{}_\mu = \frac{1}{2}\left\{h^\nu{}_\alpha\left(\frac{\partial\bar{h}_{\nu\alpha}}{\partial x^\mu} - \frac{\partial\bar{h}_{\mu\alpha}}{\partial x^\nu}\right) + \left(\bar{h}^\nu{}_\alpha\,\frac{\partial h_{\nu\alpha}}{\partial x^\mu} - h^\nu{}_\alpha\,\frac{\delta\bar{h}_{\mu\alpha}}{\delta x^\nu}\right)\right\}\gamma^\mu\,. \tag{27}$$

On substituting the value (16) for the coefficients in this expression, the quantity within the brackets $\{\ \}$ becomes

$$\chi_\mu = \frac{2\pi ie}{hc}\,\Theta_\mu + f^\nu{}_\alpha\left(\frac{\partial f_{\nu\alpha}}{\partial x^\mu} - \frac{\partial f_{\mu\alpha}}{\partial x^\nu}\right) = \frac{2\pi ie}{hc}\,\Theta_\mu + G_\mu\,, \tag{28}$$

where G_μ denotes the terms in $f^\nu{}_\alpha$ etc.

([8]) H. T. FLINT: *Proc. Roy. Soc.*, A **150**, 432 (1935).
([9]) H. T. FLINT and E. M. WILLIAMSON: *Zeits. f. Phys.*, **135**, 260 (1953).

Since the f^{ν}_{α} terms are independent of x^5

$$\chi_5 = \frac{2\pi i e}{hc}\,\Theta_5\,, \qquad G_5 \text{ being equal to zero}\,.$$

4. – The Interpretation of $K^{\mu}{}_{\mu} = 0$.

The expression G_m appears in Einstein's theory of distant parallelism and he has suggested that, multiplied by an appropriate constant, it should be identified with a component of the electromagnetic potential [4].

The suggestion was made later [7] that in its five-dimensional form it should be identified with the generalized momentum $\Pi_m = p_m + q\varphi_m/c$, where q is the charge of the particle considered, p_m a component of its momentum and φ_m a component of the electromagnetic potential of the field in which the particle is situated. q is now placed equal to $n'e$.

The same suggestion arises again here for the equation (26) in the form

$$\chi_\mu \gamma^\mu = 0$$

calls to mind the relation $\Pi_\mu \gamma^\mu = 0$, which is in a form of the mass relation $M = M_0/\sqrt{1 - v^2/c^2}$, and suggests that Π_μ is proportional to χ_μ.

A suggestion for the exact relation between Π_μ and χ_μ may be taken from a comparison of Π_5 and χ_5, that is to say, of Π_5 and $(2\pi i e/hc)\Theta_5$, since $G_5 = 0$.

From the assumption concerning the dependence of all functions upon x^5, $e\Theta_5/hc$ is placed equal to $1/l_0$, where $l_0 = h/m_0 c$.

Thus

$$\Pi_5 = n' m_0 c = n' e \Theta_5/c$$

and

$$\frac{2\pi i}{h}\,\Pi_5 = n'\chi_5\,.$$

The identification

$$\frac{2\pi i}{h}\,\Pi_\mu = n'\chi_\mu \tag{29}$$

is thus adopted. But Π_μ is a real quantity and the relation requires that G_μ should vanish. It has been seen already that $G_5 = 0$, so that the further condition $G_m = 0$ must be satisfied.

With the values given in equations (23) and (24) the expression for G_m may

be reduced to

$$f^n{}_a\left(\frac{\partial f_{na}}{\partial x^m} - \frac{\partial f_{ma}}{\partial x^n}\right),$$

and this must vanish for the four values of m.

The expression contains no suffix 5 and is a covariant four-dimensional expression.

An interesting way of providing for the condition is to adopt the relation:

$$\text{(30)} \qquad \operatorname{div}(f^n{}_a) = \frac{\partial f^n{}_a}{\partial x^n} + \Gamma^n_{ln} f^l{}_a = 0 \,,$$

where Γ^n_{ln} is the four-dimensional bracket expression. In terms of the coefficients $(f^\nu{}_\alpha)$,

$$\Gamma^n_{ln} = f^n{}_a \frac{\delta f_{na}}{\delta x^l} \,.$$

It follows that

$$f_{ma} \operatorname{div} f^n{}_a = f^n{}_a\left(\frac{\partial f_{na}}{\partial x^m} - \frac{\partial f_{ma}}{\partial x^n}\right),$$

so that if the relation (30) be adopted the condition $G_m = 0$ follows.

In this case from (28) and (29)

$$\text{(31)} \qquad \Pi_m = n'e\,\Theta_m/c$$

and

$$\text{(32)} \qquad p_m = n'e(\Theta_m - \varphi_m)/c$$

$$\text{(33)} \qquad p^m = n'eg^{mn}(\Theta_n - \varphi_n)/c \,.$$

Θ_m has been introduced as a component of the five-vector but from (32) it can be regarded also as a component of a four-vector. It is necessary to distinguish between $\gamma^{\mu\nu}\Theta_\nu$ and $g^{mn}\Theta_n$, the former being a five- and the latter a four-vector. To avoid confusion place

$$\psi_m = \Theta_m \,, \qquad \psi^m = g^{mn}\psi_n \,.$$

Thus

$$\text{(34)} \qquad p^m = n'e(\psi^m - \varphi^m)/c \,, \qquad p_m = n'e(\psi_m - \varphi_m)/c$$

and the identifications are expressed in a four-dimensional form, the mo-

mentum (p_m) being identified with the field quantities ψ_m and φ_m. If at any point of the continuum a charge $n'e$ is situated its momentum is given by the equations (34). From these equations, since $p^m p_m = -n^2 m_0^2 c^2$, it appears that

$$(\psi^m - \varphi^m)(\psi_m - \varphi_m) + (n m_0 c^2/n'e)^2 = 0 \,. \tag{35}$$

This is similar in form to Dirac's relation ([11])

$$\varphi^m \varphi_m + m_0^2 c^4/e^2 = 0$$

applicable to an electron. It is interesting that in the present theory the vector $(\psi^m - \varphi^m)$ is given an absolute value and not the vector (φ^m). (φ_m) need not lose its property of gauge invariance provided that the addition of a gradient of a scalar quantity to it is accompanied by a subtraction from ψ_m.

A suggestion for the association of the momentum of a particle with the field in which it is situated has already been made by W. SCHERRER ([10]) but his proposal is that $p_m = -n'e\varphi_m/c$. This leads to the difficulty that the generalized momentum is always zero. Although SCHERRER does not make use of a system of orthogonal axes his notation, which applies to a four-dimensional continuum is substantially the same as that used here. According to (32) the equation of motion of the particle is

$$\frac{\mathrm{d}p_m}{\mathrm{d}\tau} = \frac{n'e}{c} \frac{\partial}{\partial x^n} (\psi_m - \varphi_m) \frac{\mathrm{d}x^n}{\mathrm{d}\tau} \,.$$

If the case when there is no gravitational field be considered, by applying the relation (35), it follows that

$$(\psi_n - \varphi_n) \frac{\partial}{\partial x_m} (\psi_n - \varphi_n) = 0 \,.$$

By means of (34) it appears that

$$\frac{\mathrm{d}p_m}{\mathrm{d}\tau} = \frac{n'e}{c} (\varphi_{mn} - \psi_{mn}) \frac{\mathrm{d}x^n}{\mathrm{d}\tau} \,,$$

where $\varphi_{mn} = \partial\varphi_n/\partial x^m - \partial\varphi_m/\partial x^n$ and ψ_{mn} is a similar expression in ψ_m and ψ_n.

This is the familiar form of the equation of motion of a charged particle in an electromagnetic field, but to make it agree with the equation of Lorentz it would be necessary to place $\psi_{mn}(\mathrm{d}x^n/\mathrm{d}\tau)$ equal to zero.

([10]) W. SCHERRER: *Zeits. f. Phys.*, **138**, 16 (1955).

([11]) P. A. M. DIRAC: *Proc. Roy. Soc.*, A **209**, 291 (1951); B. HOFFMANN: *Phys. Rev.*, **87**, 703 (1952).

5. – The theory of a charged particle.

The function (ψ^m), which makes its appearance through the components $(h^\nu{}_\alpha)$ characteristic of the geometry of the continuum, has been treated as analogous to the electromagnetic vector. It would thus be expected to satisfy certain field equations. It has generally been assumed that the equations of the field of an electric charge are of the Maxwell type and this will be assumed here So far a particle situated in an external field has been considered and the vectors (ψ^m) and (φ^m) have been assumed to exist independently of it. The problem now is to relate (ψ^m) and (φ^m) to the charge regarded as responsible for their existence. The assumption that will be made is that the same relation between momentum and field exists in this case as in the case which has been considered. The charge $n'e$ now refers to what may be described as the field generating charge and it thus follows from equations (34) that the velocity of the particle satisfies the relation

$$\frac{\mathrm{d}x^m}{\mathrm{d}\tau} = \frac{n'e}{nm_0c}(\psi^m - \varphi^m)\,. \tag{36}$$

If it be assumed that the charge extends over a region of linear dimensions $r_0 = e^2/m_0c^2$ i.e. over a volume $f^2r_3^0$, where f is a constant, the courrent-density vector is

$$J^m = \frac{n'e}{f^2r_0^3c}\frac{\mathrm{d}x^m}{\mathrm{d}\tau} = \frac{n'^2(\psi^m - \varphi^m)}{nf^2r_0^2}\,. \tag{37}$$

According to Maxwell's equations it is to be expected that

$$\frac{\partial\varphi^{mn}}{\partial x^n} = J^m\,.$$

If the case $n = n' = 1$ be considered

$$\frac{\partial\varphi^{mn}}{\partial x^n} = k^2(\psi^m - \varphi^m)\,, \tag{38}$$

with $k = 1/fr_0$.

The vector (ψ^m) is the new feature introduced in this theory and it is desirable to impose upon it as simple a condition as possible without introducing any other new feature.

A simple field equation made in analogy with Maxwell's equations is

$$\frac{\partial\psi^{mn}}{\partial x^n} = 0\,. \tag{39}$$

The equations (38) and (39) are adopted as the field equations satisfied by (ψ^m) and (φ^m) and the analogies which have been used as a guide to their form suggest that they can be regarded also as the equations of the electron.

It may be noted that if ψ^m be placed equal to $\sigma\varphi^m$, there σ is some function of the coordinates and if $k^2(\sigma - 1)$ be placed equal to λ, the equation (38) becomes

$$\frac{\partial\varphi^{mn}}{\partial x^n} = \lambda\varphi^m ,$$

which has been proposed by DIRAC in his theory of the electron ([11]). The forms (38) and (39) have also been proposed, as the basis of a theory of the electron, by F. BOPP ([12]), who deduced them from a Lagrangian function differing from familiar forms in its dependence upon derivatives of the field intensities. He has shown that in the static case a solution for φ $(= - i\varphi_4)$ is given by

$$\varphi = e(1 - e^{-kr})/r ,$$

where k according to the present theory is equal to $1/fr_0$. Bopp's calculation places $f = \frac{1}{2}$. It is to be noted that φ has no singularity in the limit $r \to 0$ but attains the value $ke = 2/\alpha$. As r increases from the value zero the potential rapidly approaches the Coulomb value.

A form of these equations of a more general type in the notation of this discussion, originally proposed for the theory of the electron ([13]), is

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = S^\mu$$

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu} ,$$

where (A_μ) denotes a vector potential, corresponding to (Θ_μ). They can readily be written in four-dimensional form by using the values of the coefficients given in equations (4). They were later used by YUKAWA in his theory of the vector meson. The equation now proposed are a simplification of these more general forms.

([12]) F. BOPP: *Ann. der Phys.*, **38**, 345 (1940); H. T. FLINT and E. M. WILLIAMSON: *Nuovo Cimento*, **11**, 188 (1954).

([13]) J. W. FISHER and H. T. FLINT: *Proc. Roy. Soc.*, A **126**, 644 (1929).

RIASSUNTO (*)

Si sviluppa una teoria delle particelle cariche estendendo il concetto degli assi locali espresso da EINSTEIN. Si aggiunge al sistema geometrico un nuovo quadrivettore in aggiunta al vettore potenziale elettromagnetico. La particella si identifica quindi per mezzo della geometria e la discussione conduce a una teoria relativistica dell'elettrone nella quale sono evitate le difficoltà che scaturiscono dalla considerazione della autoenergia.

(*) *Traduzione a cura della Redazione.*

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Angular Correlations in V° Type Decays.

M. Deutschmann

Max-Planck-Institut für Physik - Göttingen

M. Cresti, W. D. B. Greening, L. Guerriero, A. Loria and G. Zago

Istituto di Fisica dell'Università - Padova
Istituto Nazionale di Fisica Nucleare - Sezione di Padova

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Summary. — Measurements have been made on 34 V^0-events produced by cosmic ray particles in the lead plates of a cloud chamber, and the angular correlation between the production and decay planes has been studied. If one limits the analysis to the V^0-particles produced in interactions in which only a small number N_h of heavily ionizing particles is seen to be emitted, a correlation is found in favour of small angles, whereas for large N_h the correlation appears to vanish.

1. – Introduction.

It is well known that a study of angular correlations may help to shed some light on the spins of the new, heavy, unstable particles. Even though it is now generally believed that the metastability of these particles can be explained by selection rules of the type proposed by Gell-Mann and Pais (1) instead of by the very high spins previously proposed by Fermi and Feynman (2), a knowledge of their actual spins is nevertheless of considerable importance.

(1) M. Gell-Mann and A. Pais: *Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics*; M. Gell-Mann: *Conferenza Internazionale sulle Particelle Elementari, Pisa* (1955); to be published in *Suppl. Nuovo Cimento.*

(2) E. Fermi and R. P. Feynman: as cited by M. Gell-Mann and A. Pais, see (1).

Workers at Brookhaven ([3]) have studied V^0-particles produced in hydrogen by the pion beam of the Cosmotron, and have found indications of a spin greater than $\frac{1}{2}\hbar$ for the Λ^0 (*). However, in various experiments ([4-7]) in which the V^0-particles were produced in heavy nuclei by cosmic rays, no clear evidence has yet been forthcoming. It has been suggested ([5]) that scattering within the nucleus may account for this difference.

In an earlier paper ([8]) we have given preliminary results of a study of angular correlations which was based on 24 events. We have now found a total of 34 events and our new statistics have confirmed our previous conclusions; final results are given in the present paper.

2. – Experimental procedure.

All the events have been found in a multiplate chamber having useful dimensions of 70×70 cm with an illuminated depth of 20 cm. Three different arrangements of lead plates have been used: (*a*) 2 cm thick plates with 6 cm of free space between the plates, (*b*) 1.5 cm thickness and 6 cm of free space, and (*c*) 1 cm thickness and 3 cm of free space: most of the events were obtained while using arrangements (*b*) and (*c*). The events have been reconstructed by reprojecting the two stereoscopic photographs onto a fixed plane corresponding to the front of the lead plates. The actual positions and angles of the tracks were then calculated from the projected positions and angles by means of a purely analytical procedure ([+]). The correlation looked for was that between the plane containing the incoming primary and the outgoing V^0-particle (production plane) and that containing the two decay products (decay plane).

In general, the only useful events were those in which the primaries were ionizing, but to these have been added events produced by neutral particles

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([3]) W. B. Fowler, R. P. Shutt, A. M. Thorndyke and W. L. Whittemore: *Phys. Rev.*, **98**, 121 (1955).

(*) *Note added in proof.* – See also recent work by W. D. Walker and W. D. Shepard: in course of publication.

([4]) J. Ballam, A. L. Hodson, W. Martin, R. Rau, G. T. Reynolds and S. B Treiman: *Phys. Rev.*, **97**, 245 (1955).

([5]) J. D. Sorrels: *Proceedings of the Fifth Annual Rochester Conference* (1955).

([6]) B. Rossi: *Proceedings of the Fifth Annual Rochester Conference* (1955).

([7]) D. B. Gayther and C. C. Butler: *Phil. Mag.*, **46**, 467 (1955).

([8]) M. Deutschmann, M. Cresti, W. B. D. Greening, L. Guerriero, A. Loria and G. Zago: *Conferenza Internazionale sulle Particelle Elementari, Pisa* (1955); to be published in *Suppl. Nuovo Cimento*.

([+]) By using the G1 Electronic Computer of the Max-Planck-Institut für Physik at Göttingen.

TABLE I

Event No.	Nature of particle	p_0 MeV/c	α	β	γ	φ	Primary	N_h	n_s	E_{π^0} GeV	Notes
a2445	Λ^0	400	15° ± 15°	−19°	−33°	47°	s	2	8	2.5	
a2605	Λ^0	970	2° ± 10°	58°	58°	89°	$l(n)$	0	2?	?	
a2997	θ^0	880	40° ± 20°	20°	20°	20°	$l(n)$	2	2	0	
a7393	θ^0	1210	67° ± 18°	−71°	4°	17°	l	0	0	0	
a7842	Λ^0	310	86° ± 12°	− 5°	66°	62°	s	5	6	1.0	
b8959	?	—	45° ± 20°	81°	−55°	14°	s	0	1	0.5	
b1530	Λ^0	590	24° ± 10°	58°	41°	44°	s	1	6	0.5	
b7655	θ^0	750	30° ± 6°	19°	− 4°	24°	s	0	0	1.5	
b8313	Λ^0	510	48° ± 18°	−15°	−12°	89°	s	3	9	2	
b9146	Λ^0	202	31° ± 4°	−78°	71°	68°	s	3	4	0.4	
b9279	θ^0	900	34° ± 13°	−14°	18°	26°	s	2	1	0.5	
b0488	θ^0	730	53° ± 3°	24°	73°	36°	s	2	9	0.5	
b2565	Λ^0	380	12° ± 5°	−31°	−19°	27°	l	0	0	0	
b4458	θ^0	470	14° ± 5°	−65°	−77°	36°	$s(n)$	0	4	2.5	
b4585	θ^0	360	14° ± 14°	28°	42°	85°	l	1	1	0	
b8067	θ^0	1760	6° ± 12°	33°	35°	22°	s	0	4	6.0	
b8571	Λ^0	470	12° ± 4°	49°	37°	34°	s	5	7	13	
b8665	Λ^0	300	48ν ± 11°	47°	6°	127°	$s(n)$	0	7	2.5	
b8939	θ^0	380	76° ± 10°	−13°	53°	49°	$s(n)$	2	9	3.5	
b0782	Λ^0	880	41° ± 10°	70°	34°	20°	$s(n)$	0	0	14	
b1205	Λ^0	500	3° ± 8°	−68°	−44°	19°	l	5	1	0.5	
b1833	θ^0	390	10° ± 12°	−10°	− 8°	81°	s	5	2	0.2	
c3221	θ^0?	670	44° ± 9°	−44°	−76°	40°	l	1	1	0	
c3269	θ^0	2050	12° ± 9°	−60°	−72°	35°	l	0	0	0	
c4135	Λ^0	590	11° ± 15°	−35°	−28°	66°	l	1	2	0.3	
c4457	?	—	64° ± 20°	30°	−22°	36°	l	4	3	0	
c8777a	Λ^0	730	11° ± 20°	4°	14°	41°	s	0	1	0	associated
c8777b	θ^0	1430	3° ± 20°	26°	25°	27°	s	0	1	0	production
c9581	θ^0	590	10° ± 10°	26°	33°	88°	l	0	0	0.3	
c0136	?	—	16° ± 15°	−47°	−54°	56°	l	0	0	0	
c1278a	θ^0?	990	47° ± 7°	65°	−61°	37°	s	2	1	0.2	associated
c1278b	θ^0	1600	90° ± 5°	−22°	10°	15°	s	2	1	0.2	production
c1858	Λ^0	320	26° ± 3°	48°	75°	21°	l	0	1	0	
c3761	Λ^0	310	49° ± 12°	−60°	−48°	107°	s	3	4	0	

p_0 Momentum of the V^0 particle;
α Angle between the production and decay planes;
β Azimuthal angle between the normal to the production plane and the normal to the front plate of the chamber;
γ as above, but for the decay plane instead of for the production plane.
φ Emission angle of the V^0 particle with respect to the direction of the primary;
s Primary particle arriving singly;
l Primary particle locally produced;
(n) Neutral primary particle;
N_h Number of heavily ionizing particles observed in the parent interaction of the V^0 particle;
n_s Number of penetrating shower particles;
E_{π^0} Energy of the photoelectronic component.
Note. - The prefixes a, b, and c in column 1 indicate the arrangement of Pb plates used.

when the uncertainty over their directions did not lead to an excessive error in calculating the production plane. For instance, in certain types of event one may assume that the neutral primary had a common origin with other particles seen in the photograph, or, if it produced a well collimated shower, its direction may be taken as that of the axis of the shower. In practice we have considered only those events for which the error in the correlation angle is not greater than 20°.

3. – Results and discussion.

The 34 events are presented in Table I together with all the relevant data. From this may be obtained the frequency distribution of the angle α between the production and the decay planes which is given in Fig. 1 for all the events. Most of the V^0-particles have been recognized as either Λ^0 or θ^0 (see column 2 of Table I); in some cases the identification is less certain, while three events are unidentifiable. The separate frequency distributions for these types of particle will be seen from Table II. There appears to be a small-angle cor-

TABLE II.

Angular interval	Λ^0	θ^0	Unidentified
$0° \div 29°$	9	7	1
$30° \div 59°$	5	6	1
$60° \div 90°$	1	3	1

relation which is a little more pronounced for the Λ^0 then for the θ^0. However, before we can draw any conclusions we must see if our experimental results represent a genuine correlation or not.

It would be possible, for instance, to think of an experimental bias which would reduce the frequency at large values of α. Such a bias might be produced by the combination of two facts. First, the most favourable position for seeing a V^0-decay is when the plane containing the decay products roughly corresponds to that of the chamber; this is especially important when the V^0 is associated with a dense shower. Second, due to the relative shallowness of the chamber, primary particles coming in from the front or from the back are more easily lost than are those which arrive nearly parallel to the plane of the chamber. Together these two effects may tend to reduce the frequency at large values of α.

We have examined this point (for the total sample of 34 events), and as a measure of the orientation of the production and decay planes, we have

considered the azimuthal angles β and γ which the normals to these two planes make with respect to that of the front plate of the chamber. As will be seen from columns 2 and 3 of Table III, small effects of the type anticipated are

TABLE III.

Angular interval	β	γ	$\lvert\beta-\gamma\rvert$	Bias
$0^\circ \div 29^\circ$	14	12	21	12.0
$30^\circ \div 59^\circ$	11	14	10	11.1
$60^\circ \div 90^\circ$	9	8	3	10.9

present, and it is possible that together these may lead to a fictitious correlation, even when a genuine correlation is completely absent. The absence of a genuine correlation would imply that the production and decay planes are completely independent of one another. Thus if we successively combine each production plane with all of the decay planes we will destroy any true correlation and so obtain a distribution which has only been influenced by the bias. Since this calculation would be very tedious if one used the angle α, we have preferred to use the difference $\lvert\beta-\gamma\rvert$. We think that this is justified for the calculation of the bias since the distribution of $\lvert\beta-\gamma\rvert$ (column 4, Table III) is similar to the distribution of α (Fig. 1). The fictitious correlation is then given in column 5, normalized to 34 events. We interpret these figures as meaning that the bias is negligible. In addition it would be difficult to explain how an instrumental bias could have acted more strongly on the Λ^0 then on the θ^0-particles (Table II).

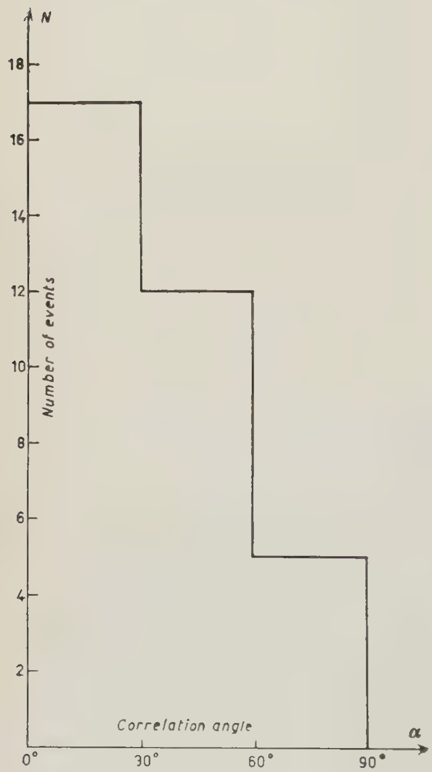

Fig. 1.

If we accept that systematic effects are not present; we must consider the possibility that in Fig. 1 we are dealing with a statistical fluctuation from a truly isotropic distribution. In order to calculate the probability for such a fluctuation, we have made use of the deviation of the measured mean values,

$\bar{\alpha}_{\text{meas}}$, from the corresponding mean value for an isotropic distribution, $\bar{\alpha}_{\text{iso}}$, $= 45°$. It can be shown that, in the case of a random sample of 34 events which form part of an isotropically distributed population, their mean value $\bar{\alpha}$ has a standard deviation from 45° given by $\mu = \pm 4.45°$. For our 34 events the measured mean value is $\bar{\alpha}_{\text{meas}} = 32.2°$, and so:

$$\Delta\bar{\alpha} = \bar{\alpha}_{\text{iso}} - \bar{\alpha}_{\text{meas}} = 12.8° = 2.87\,\mu\,.$$

Now the probability of finding a deviation of at least ± 2.87 times the standard deviation is only 0.4%; thus our sample of events is in favour of a genuine small-angle correlation. On the other hand this result is somewhat different from the findings of other groups who observed similar numbers of V^0-decays under similar conditions, and who found a less marked maximum at small angles (6,7) or no maximum at all (5). We therefore think that only richer statistics will permit definite conclusions about the presence or absence of spins for the Λ^0 and θ^0-particles. Further, as already mentioned, statistics based on V^0-particles produced in heavy nuclei may be greatly influenced by a « smearing out » effect due to secondary interactions of the V^0-particles within the nuclei in which they are produced, and the measured angular correlations may therefore result to be too flat. We will now present some evidence for the existence of such an effect.

If scattering of the V^0-particles reduces a pre-existing angular correlation, then it ought to be possible to find differences (even for V^0-particles produced in homogeneous materials such as lead) if one distinguishes between interactions involving only a small part of the nucleus and those which involve a large part of it; this distinction may be based on the number of heavily ionizing particles which are produced in the same interaction. It would be logical to suppose that in a glancing collision the number N_h of heavily ionizing particles would be small, whereas in a central collision it would be large. We have therefore divided the events into two groups having respectively $N_h \leqslant 1$ and $N_h \geqslant 2$ (*). It will be seen that for $N_h \leqslant 1$ the small-angle correlation appears to be present, but that for $N_h \geqslant 2$ this is not so. In Table IV the events have been further sub-divided into Λ^0 and θ^0. Naturally, their numbers have now become too small to permit reliable conclusions, but this sub-division will at least serve to demonstrate that separation according to

(*) Particles have been considered as heavily ionizing if their ionization was at least three times minimum. Naturally the observed number is smaller then the total number of heavily ionizing particles emitted from the nucleus since most of them cannot leave the lead plates. For plates 1.5 cm thick, about one tenth is expected to be visible (9).

(9) M. DEUTSCHMANN: *Zeits. f. Naturf.*, 9a, 477 (1954).

TABLE IV.

Angular interval	$N \leqslant 1$			$N_h \geqslant 2$		
	All events	Λ^0	θ^0	All events	Λ^0	θ^0
$0^\circ \div 29^\circ$	13	6	6	4	3	1
$30^\circ \div 59^\circ$	5	2	2	7	3	4
$60^\circ \div 90^\circ$	1	0	1	4	1	2

N_h does not automatically correspond to a separation into Λ^0 and θ^0 (which would be the case if, for example, the Λ^0 were associated with smaller N_h than the θ^0). The dependence on N_h indicates that the observed correlations are affected by collisions of the V^0 within the nucleus (*).

In addition we have tried to find a dependence of the angular correlation upon various other quantities. For example, the correlation appears to be somewhat different for low- and for high-momentum V^0-particles; similarly it seems to be slightly different for small and for large emission angles φ of the V^0. However, since both the momenta and the emission angles are, on the average, different for the Λ^0 and for the θ^0, these divisions lead automatically to a partial separation into the two types of particle, which, as seen from Table II, show different degrees of correlation. Therefore it is not possible to decide if, apart from the dependence on the type of particle, there is an additional dependence on the momentum or on the emission angle.

We have also investigated the possibility of a connection between the angular correlation and (i) the number n_s of penetrating shower particles seen to come from the parent interaction of the V^0, (ii) the energy E_{π^0} of the photo-electronic component of the parent shower, and (iii) the nature of the primary particle, i.e. if arriving singly or if locally produced. None of these quantities appears to have an influence on the correlation.

4. – Conclusions.

In a study of 34 V^0-events produced in lead, we have found a correlation between the production and decay planes which favours small angles. This correlation is more marked if one considers only those V^0-particles which have been produced in interactions of a more « elementary » type, i.e. in inter-

(*) Scattering of V^0-particles in heavy nuclei has also been postulated by JAMES and SALMERON [*Phil. Mag.*, **46**, 571 (1955)] as a result of their investigations into the dynamics of double V^0-events.

actions which involve only one or a few nucleons of the target nucleus, and which consequently show a small number of heavily ionizing tracks. On the other hand, if many heavily ionizing particles are emitted together with the V^0-particle, no correlation is found. We therefore conclude that V^0-particles undergo collisions in the parent nuclei which can destroy an existing orientation of these particles. Consequently one is led to suppose that negative results obtained in studies of directional effects in the decay of heavy unstable particles which were produced in heavy elements such as lead, copper or photo-emulsion, cannot yet be regarded as definite evidence against a spin of these particles.

* * *

We would like to thank Professor A. Rostagni for his support, and one of us (M.D.) has to thank Professor W. Heisenberg for discussions and for permission to work in Italy. We are very pleased to acknowledge our indebtedness to the Società Adriatica di Elettricità (S.A.D.E.), and in particular to Ingegner M. Mainardis and Ingegner M. Pancini, for assistance which made possible the work at the Laboratorio della Marmolada.

RIASSUNTO

Si sono fatte misure su 34 eventi V^0 prodotti da particelle della radiazione cosmica nei setti di piombo di una camera di Wilson, e si è studiata la correlazione fra il piano di produzione ed il piano di decadimento di tali eventi. Limitando l'analisi alle V^0 prodotte in interazioni in cui appare essere emesso solo un piccolo numero N_h di particelle ad alta ionizzazione, si trova una correlazione verso i piccoli angoli, mentre per grandi valori di N_h sembra che la correlazione sparisca.

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On the Spin of Artificially Produced τ-Mesons,

B. BHOWMIK (*), D. EVANS, I. J. VAN HEERDEN (+) and D. J. PROWSE
H. H. Wills Physical Laboratory, University of Bristol

(ricevuto il 2 Gennaio 1956)

Summary. — Measurements have been carried out on the π-meson secondaries arising from the decay of positive τ-mesons recorded in a stack of nuclear emulsions which had been exposed to the positive K-meson beam of the Berkeley « bevatron ». The distribution in energy of these secondaries is shown to be consistent with the view that the τ-meson is a pseudo-scalar particle and they appear to exclude the possibility that the spin and parity of the particle are 1^-, 1^+, 2^+ or 3^-.

1. – Introduction.

DALITZ (1) and FABRI (2) have shown that information about the spin and parity of the τ-meson can be obtained by studying the energy distributions of the secondary π-mesons. It is usual to describe the dynamics of the τ-meson decay by two parameters: the energy, E, of the secondary meson of unlike charge and $\cos\theta$, where θ is the angle at the instant of decay between the direction of the unlike π-meson and that of the two like mesons in their own c.m. system. The quantity $\cos\theta$ is related to the way in which the available energy is divided between the two π-mesons of like charge.

On the assumption that the dimensions of the τ-meson are small compared with the wavelengths of the emitted π-mesons, five spin and parity combinations give unique distributions for E and $\cos\theta$. These five cases are:

(*) On leave from the University of Delhi.
(+) On leave from the National Physical Laboratory, Pretoria.
(1) R. H. DALITZ: *Phys. Rev.*, **94**, 1046 (1954).
(2) E. FABRI: *Nuovo Cimento*, **11**, 479 (1954).

0−, 1+, 1−, 2+ and 3−. Other possible combinations of spin and parity (e.g. 2−, 3+, 4−, ...) yield distributions of E and $\cos\theta$ which are superpositions of two or more curves whose relative amplitudes are unknown because of their dependence on knowledge of the τ-meson structure. Only a detailed correlation between the experimental data and the possible theoretical distributions of E and $\cos\theta$ would make it possible to differentiate between these higher spin values.

Previous work based largely on the τ-mesons found in nuclear emulsions exposed to cosmic radiation has been summarized by AMALDI [3]. This analysis has been carried out on events found in various stacks of different sizes, and is subject to a number of difficulties. Thus, the efficiency for stopping the unlike π-meson secondary as a function of its energy, is not known; effects in the distribution of E and $\cos\theta$ due to observational biases, will therefore remain undetected. It is also uncertain whether all the τ-mesons produced by the cosmic radiation are positively charged. This makes it necessary to stop two of the secondary π-mesons of like charge in order to determine the energy of the secondary of unlike charge.

The previous results support a pseudo-scalar particle theory for the τ-meson. It would, therefore, appear that this particle is different from the θ^0 or χ-mesons which cannot have even spin and odd parity, because they decay into two π-mesons. Recent work on K-mesons indicates that their mean lifetimes and their masses are constants which are independent of the mode of decay. This gives some justification for the view that there is a single type of K-particle which can decay in several different modes. It becomes important, therefore, to obtain further evidence on the spin and parity of the τ-meson. The present experiments were made in the favourable conditions provided by allowing a beam of artificially produced K^+-mesons to enter a stack of emulsions.

2. – **Experimental Procedure.**

The stack used in this experiment consisted of 120 sheets of emulsion, each 16″ × 10″ × 600 μm exposed to the magnetically focussed K^+-meson beam from the Bevatron. The size of the stack was primarily designed to arrest a considerable proportion of the fast secondaries from the K_{μ_2} and K_{μ_3} modes of decay.

Only the first thirty plates were available in Bristol; the first twenty-five were area-scanned in a region where the K-mesons were known to stop and this scan yielded ninety-three τ-mesons decaying at rest. Plates 20-25 were scanned in Dublin, and for this we are indebted to Professor O'CEALLAIGH.

(3) E. AMALDI: *Provisional Report of the Pisa Conference*, 1955.

The fraction of negative π-mesons expected to stop in the thirty plates used in the experiment has been calculated (see Appendix I) and is shown as a function of energy in Fig. 1. As can be seen, few high energy π-mesons will be arrested; this results in a reduction of the sensitivity of the experiment to the detailed shape of the distribution at high energies. In the analysis, we have only used those τ-mesons for which the negative π-meson has been stopped. For the five simple cases of spin and parity, the form of the $\cos\theta$ distribution is independent of energy. The $\cos\theta$ distribution is therefore unaffected by the geometrical loss factor and discriminates between curves which differ only at high energies.

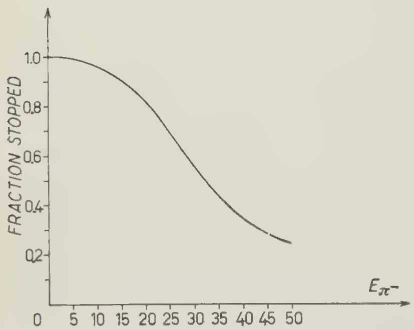

Fig. 1. – The fraction of negative π-mesons expected to stop as a function of energy.

2'1. *Methods of Measurements.* – The secondary π-mesons were required to be co-planar to within 1° to exclude decays in flight and cases in which a γ-ray is also emitted. Where possible, the energies of the π-mesons were determined from their ranges; these were measured with a stage micrometer and, in the absence of a stopping power calibration, the range-energy relation of BARONI *et al.* [4], was used. The emulsions are packed by the manufacturers at a constant humidity and it is unlikely that it differed from this value when the exposure was made. In all, we do not expect a systematic error in our range measurements of more than 2%, or a « straggle » of more than 5%.

In those cases where two π-meson energies were known from ranges, that of the third was deduced using an assumed Q-value of 75.0 MeV. It might be expected, therefore, that any set of π-meson energies, calculated entirely from range, would have a maximum uncertainty of $\sim 4\%$. When both positive π-mesons left the stack and only the negative one was stopped, their energies were calculated from the relative space angles. The projected angles in the emulsion plane were measured to $\pm 0.1°$ by means of an ocular protractor. In addition, for all events the angles of dip were measured on the same microscope; the fine focus of this instrument was graduated in 0.1 μm divisions and this was the accuracy with which the microscope could be focussed and the depth of a grain determined. The angle of dip in unprocessed emulsion was obtained by assuming an original thickness of 600 μm.

(4) G. BARONI, C. CASTAGNOLI, G. CORTINI, G. FRANZINETTI and A. MANFREDINI: *CERN Bulletin*, BS9 (1954).

The effect of small experimental errors on the energies calculated from space angles are extremely difficult to compute and in fact no generalised expresssion can be given. We have, therefore, considered experimentally the differences between the energies when they have been calculated both from residual range and from space angle measurements.

The non-relativistic energy of any secondary is given by:

$$E = \frac{Q \sin^2 \alpha}{\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma},$$

where α is the angle opposite the secondary, and β and γ are the other two space angles. It can be seen therefore, that the error in E caused by an error $\delta\alpha$ in α, is proportional to $\cot \alpha\, \delta\alpha$. This is relatively large when α approaches π and will occur when the secondary is of very low energy. Fortunately, it is of course, never necessary to determine the energies of slow secondaries in this way; they will always be arrested in the stack and their energy can be determined from the residual ranges. Fig. 2 shows a histogram of the values of $(E_\theta - E_R)/E_R$ (where E_θ and E_R are energies determined from angles and range respectively) for each secondary, in those examples in which it has been possible to make a dual determination, and for which the energy is greater than 10 MeV.

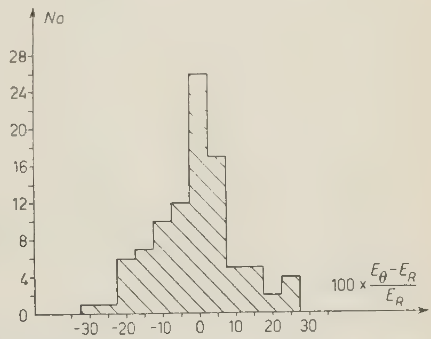

Fig. 2. – Histogram of $(E_R - E_\theta)/E_R$ for $E > 10$ MeV.

The distribution is normal and is centred on zero. This indicates that no systematic error will result from employing values of E_θ, but the observed width of the distribution shows that a standard deviation of about 9% can be expected.

It should be noted that E_R automatically takes relativistic effects into account. In our analysis we have taken all π-meson velocities to be non-relativistic, and this does introduce a slight systematic error in E_θ, but it never exceeds 4%. In general, however, the values of the parameter « cos θ » have been determined from angle measurements, and those of E_{π^-} from range. Events may thus be expected in which the maximum π^+ energy approaches 50 MeV, whereas the relativistic maximum is 48 MeV.

The negative and positive π-mesons were distinguished by the effects observed at the end of their ranges. Positive π-mesons at rest should decay and negative π-mesons should interact with the nuclei in the emulsion. All mesons decaying in the characteristic way were, therefore, designated positive.

All σ-stars were assumed to be caused by negative π-mesons. Two cases were observed of the disappearance of the meson in flight; these were assumed to be cases of charge-exchange and as this phenomenon is ~ 15 times more probable for negative mesons than for positive (PUPPI (5)) they were designated negative. A few examples only were ambiguous and in all such cases it has been possible to confirm the charge by following the other two mesons to rest.

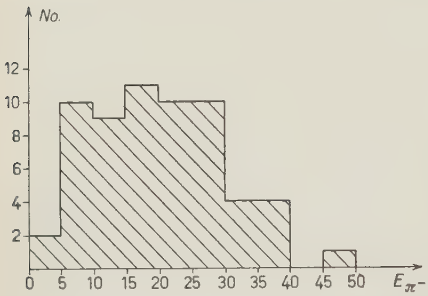

Fig. 3. – The energy distribution of the negative π-meson in 5 MeV classes.

3. – Results.

Of 96 τ-mesons found, 93 decayed at rest. One decayed in flight and in two cases, although the τ-meson appeared to come to rest, the π-meson secondaries were not co-planar; these two may therefore be examples of associated γ-ray production. Of the 93 decaying at rest, the negative meson was arrested in 61 cases; in 15 examples both positive mesons were stopped; details of these τ-mesons are summarized in Table I. In the remaining examples, it was impossible to identify the negatively charged π-meson.

Because the geometrical loss factor can readily be calculated in the cases when the negative meson is arrested, we have chosen to analyse these events separately. Fig. 3 shows the histogram obtained for the distribution of the π^--energy plotted in 5 MeV classes. In Fig. 4, theoretical curves for various spin and parity combinations are shown; these have been obtained by normalising those given by DALITZ to a total of 93 events and multiplying the resulting curve by the geometrical loss factor. The areas under these theoretical curves indicate that the numbers of negative π-mesons expected to be arrested in the stack are 61.5, 63.7, 49.8, 69.3, for $0-$, $1-$ and $3-$, $1+$, and $2+$ respectively, compared

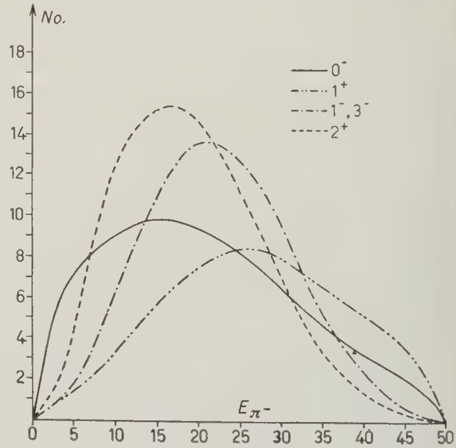

Fig. 4. – Theoretical curves for the distribution in energy from various spins and parities.

(5) G. PUPPI: *Suppl. Nuovo Cimento*, **11**, 438 (1954).

TABLE I.

No.	E_{π^-}	E_{π^+}	E_{π^+}	No.	E_{π^-}	E_{π^+}	E_{π^+}
A) τ-*mesons with* π^--*meson stopping*							
1	6.6	19.4	47.4	32	14.3	20.9	31.7
2	8.9	17.0	47.6	33	31.6	5.9	37.5
3	10.3	22.6	41.8	34	19.8	14.7	40.5
4	26.8	20.8	27.4	35	13.8	15.2	46.5
5	34.0	5.6	35.4	36	8.2	20.8	45.0
6	8.2	22.4	44.0	37	18.3	17.0	39.7
7	10.6	15.2	48.1	38	12.7	11.8	48.5
8	11.6	30.6	32.8	38	21.7	9.2	47.4
9	29.7	14.4	29.4	40	39.0	4.9	30.7
10	9.0	25.8	41.8	41	21.2	5.1	48.7
11	25.7	18.2	21.1	42	26.0	11.1	37.9
12	25.0	23.0	27.0	43	5.4	25.6	44.4
13	19.0	11.0	45.0	44	19.2	8.0	48.1
14	8.4	24.8	44.6	45	9.0	31.5	33.8
15	17.7	28.2	28.2	46	8.2	19.8	46.4
16	47.5	6.2	21.3	47	12.8	21.9	41.4
17	21.3	22.4	27.1	48	13.7	30.6	33.5
18	38.7	15.7	20.6	49	17.7	19.7	37.6
19	36.3	2.6	36.1	50	25.1	24.4	25.4
20	18.7	24.8	34.2	51	23.5	18.9	33.8
21	12.8	17.3	47.0	52	25.7	17.2	32.1
22	33.3	11.0	30.7	53	38.0	23.8	13.2
23	24.4	16.0	34.6	54	27.0	10.7	37.3
24	7.3	30.8	36.8	55	20.9	9.8	44.3
25	21.1	19.6	34.3	56	28.1	7.7	39.2
26	4.8	33.8	37.5	57	23.0	11.8	41.2
27	18.3	22.2	34.5	58	21.4	13.0	39.7
28	19.8	17.1	37.5	59	28.2	4.8	42.1
29	33.6	6.8	34.6	60	15.2	13.5	46.3
30	27.2	18.3	29.5	61	2.0	33.8	39.6
31	19.7	12.1	37.2				
B) τ-*mesons with two* π^+-*mesons stopping,* π^- *out of stack*							
62	37.6	10.4	27.0	70	26.7	11.8	36.5
63	33.4	3.6	38.0	71	22.5	15.1	37.4
64	30.8	15.0	29.3	72	45.4	13.1	16.5
65	19.1	18.5	37.4	73	28.8	5.0	41.2
66	32.9	16.5	25.6	74	30.7	19.7	24.6
67	46.6	10.1	18.3	75	42.4	3.2	29.4
68	20.0	22.0	33.0	76	39.5	5.9	29.6
69	24.8	21.7	27.0				

with the experimental number of 61. The theoretical $\cos\theta$ distributions for spins of 0—, 1+, 2+, 1—, 3— are isotropic, $\sin^2\theta$, $\sin^2\theta\cos^2\theta$ and $\sin^2\theta(5+3\cos^2\theta)$ respectively. For spins of 2—, 3+, 4—, 6—, etc., the distributions are indeterminate but can, by suitable choice of parameter, be made isotropic.

The analysis of relatively small samples is always difficult. In a χ^2-test it is essential to divide the data into groups in which at least five events are expected to occur. This necessitates using large groups when the density of expected events is low with a resultant loss of information. In our case most of the differences in the theoretical predictions occur at the extreme ends of the distributions where the expected density from our sample is low. The χ^2-test is, therefore, very insensitive to the differences we are trying to detect.

On the other hand, one can devise methods in which all the information available is used: these usually suffer by being too sensitive to extreme observations and are, therefore greatly influenced by what might seem experimentally a small error. We have, therefore chosen to analyse our results in 3 different ways. The first is the normal χ^2-test, the results of which are shown in Table II. In the first section of this table are given the values of χ^2 for the energy distribution for each theoretical curve and the corresponding Pearson probabilities. In the second section these values are given for the $\cos\theta$ distribution.

TABLE II.

Spin	(1)			(2)		
	χ^2	n	P	χ^2	n	P
0^-	3.45	5	0.63	8.70	9	0.46
1^-	13.41	5	0.02	26.60	5	< 0.001
1^+	44.20	5	< 0.001	8.70	9	0.46
2^+	4.88	5	0.43	40.13	7	< 0.001
3^-	13.41	5	0.02	29.92	7	< 0.001

The probability of observing a given distribution in E and $\cos\theta$ is given by:

$$\prod_i \{P(E_i)Q(\cos\theta_i)\}\ ,$$

where E_i and $\cos\theta_i$ are the values of the two decay parameters for the i'th τ-meson, and $P(E_i)$ and $Q(\cos\theta_i)$ the probabilities of observing the given values of E and $\cos\theta$. If we consider only those τ-mesons for which the unlike π-secondary is stopped, the above expression becomes:

$$\prod_i \{P(E_i)B(E_i)Q(\cos\theta_i)\}\ ,$$

where $P(E_i)B(E_i)$ is the probability of stopping the unlike π-meson of energy E_i. $P(E_i)B(E_i)$ can be obtained for the various spins by normalising the areas under the curves given in Fig. 3 to unity; $Q(\cos\theta_i)$ can be obtained in a similar way from the theoretical $\cos\theta$ distributions. The values of these products for the various spins are shown in Table III. It can be seen that our results are five orders of magnitude more likely to have been obtained from a spin of 0 — than for a spin of 1+. The other values being still less probable. In this analysis all the available information has been used but the products obtained are very sensitive to the values theoretically predicted for observing events with certain extreme values of E and $\cos\theta$; for example τ-mesons

TABLE III.

Spin	Probability
0^-	$5.5\cdot10^{-56}$
3^-	$2.2\cdot10^{-63}$
2^+	$2.0\cdot10^{-62}$
1^+	$2.2\cdot10^{-61}$
1^-	$1.2\cdot10^{-67}$

number 26 and 61 discriminate greatly against a spin of 1 — because of their values of E, while τ-s number 1 and 41 discriminate against a spin of 2+ because of their values of $\cos\theta$. In view of the experimental error already, quoted, therefore, the reliability of the result is open to question. It is, however, a very convenient method to use because it is cumulative: as more results are published on τ-meson analysis, the probabilities of observing the complete experimental distributions on any given spin assignment can be obtained by cumulative multiplication.

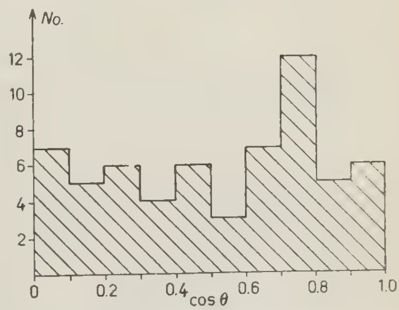

Fig. 5. – Experimental $\cos\theta$ distribution.

One can estimate the expected standard deviation on each of these probabilities. For example the expected values of $\log P(1+)/P(0-)$ for spins of 0 — and 1+ are -4.80 ± 2.3 and $+4.90\pm1.8$. Hence, the observed value of this exponent, -4.60 is about five standard deviations away from the expected value for 1+ (*).

The final method of analysis is considered to be the best in that it is not influenced to such a marked degree by the effects of experimental error, and

(*) We are indebted to Dr. R. H. DALITZ for these figures.

yet it does include a large amount of available information. The exact level of significance of the results, however, is, as in the last analysis, difficult to compute. On an empirical basis, the standard deviation on the result has been estimated to be about one order of magnitude. A convenient method of stating the τ-meson data is by plotting each τ-meson on a diagram proposed by DALITZ. Such a diagram is shown in Fig. 6 with all the τ-mesons in Class A of Table I plotted. Cos θ is the ratio between the distance of a point from the E_{π^-}-axis and the total distance from the perimeter of the circle to the axis at the given value of E_{π^+}. Bearing in mind that our estimated error on E is approximately 4% and that on cos θ about 6% we have split the diagram up into one hundred divisions, the cos θ and E scales being divided equally into ten. Given a total of 93 τ-mesons we have calculated the probability of obtaining τ-mesons in each of the 100 divisions, for each of the five spin and parity combinations. The probability of finding an event in any division remains constant for each trial and as we have a relatively large total number, the distribution is Poissonian. We have calculated from the Poissonian the probability of obtaining the observed number in each division if the average number of τ-mesons obtained in a large number of attempts is that given by each of the theoretical distributions. These probabilities have been multiplied together to obtain the total probability of observing the experimental distribution for each spin value.

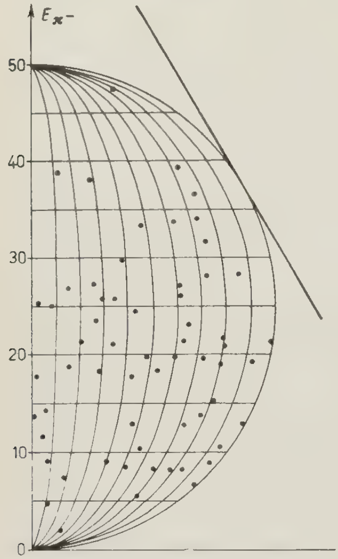

Fig. 6. – The Dalitz diagram divided into one hundred divisions with all the mesons in Class A plotted as black dots.

The results are seen in Table IV.

TABLE IV

Spin	Probability
0−	$2.1 \cdot 10^{-39}$
3−	$7.2 \cdot 10^{-45}$
2+	$5.7 \cdot 10^{-45}$
1+	$5.4 \cdot 10^{-46}$
1−	$6.3 \cdot 10^{-52}$

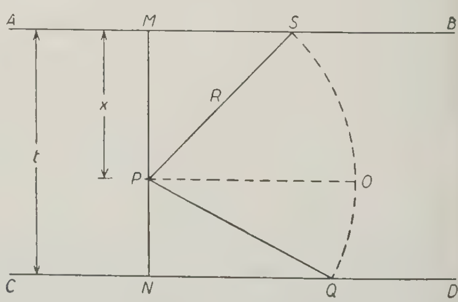

Fig. 7.

On the basis of the foregoing analysis we consider that the spins of 1 + ,

1 —, 3 —, and 2 + are excluded, this leaves 0 — as the most likely value, although we cannot exclude spins of 2 —, 3 +, 4 —, and 4 + etc.

The results obtained from τ-mesons produced by the cosmic radiation are, therefore, confirmed.

* * *

We would like to thank Professor F. C. POWELL, F.R.S., for his constant interest and encouragement; Professor C. O'CEALLAIGH for his kind permission to use the Dublin scanning data: and Dr. R. H. DALITZ for his helpful discussions and advice. We would also like to thank Dr. R. W. BIRGE and his co-workers at the Bevatron for exposing the stack. One of us (D.E.) thanks the Bristol University Scholarship Committee for a maintainance grant.

APPENDIX I

In Fig. 8, $ABDC$ represents the cross-section of a stack of nuclear emulsions of total thickness, t, with the other dimensions infinite. Let us consider the decay of τ-mesons found in a plane PO, at a distance x from the top of the stack. Of the negative π-mesons resulting from τ-mesons decaying at P with

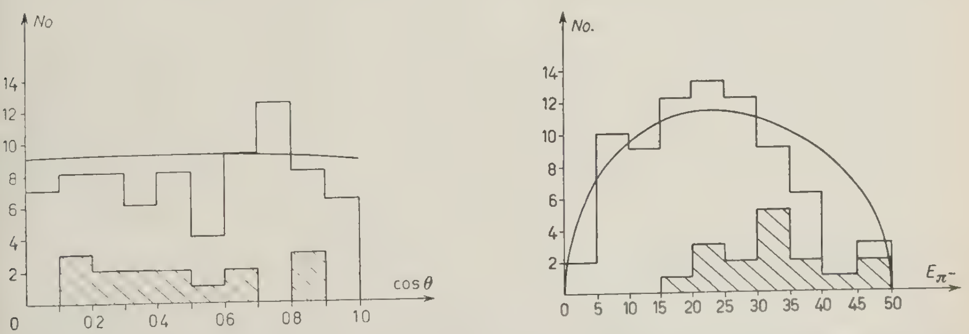

Fig. 8. – Experimental E_{π^-} and $\cos\theta$ distributions for all mesons in Table I. The shaded areas indicate those mesons for which the two π^+-mesons were arrested. The theoretical θ curves corrected for the probability of identifying the π^--meson, are also shown.

a residual range R, only those emitted within the solid angle formed by the revolution of SPD about a vertical axis MPN will be arrested in the stack. The fraction stopped is therefore equal to the ratio between this solid angle and the total sphere.

Considering the upper hemisphere:

$$\text{the solid angle of revolution} = \frac{2\pi x}{R}\,, \qquad \text{if } R > x\,,$$
$$= 2\pi\,, \qquad \text{if } R \leqslant x\,.$$

For the lower hemisphere, the solid angle of revolution

$$= \frac{2\pi(t-x)}{R}\,, \quad \text{if } R > (t-x),$$
$$= 2\pi\,, \qquad \text{if } R \leqslant (t-x).$$

A composite expression for the fraction of mesons of range R stopped would be:

$$f_{R,x} = \frac{x}{2R}\,\delta(>x) + \frac{1}{2}\,\delta(\leqslant x) + \frac{t-x}{2R}\,\delta(>\overline{t-x}) + \frac{1}{2}\,\delta(\leqslant \overline{t-x})\,,$$

where $\delta(>x) = 1$, if $R > x$ and zero otherwise,

and $\delta(\leqslant x) = 1$, if $R \leqslant x$ and zero otherwise,

and similarly for $\delta(>\overline{t-x})$ and $\delta(\leqslant \overline{t-x})$.

As we are considering the area of the plates to be infinite all points P in the plane PO are equivalent and this composite expression holds for all secondaries emitted from the τ-mesons found in plane PO.

For a given plate, r, f_{R,x_r} can now be evaluated where x_r is the distance from the top of the stack to the centre of the plate. Suppose the number of τ-mesons decaying in plate r, to be n_r and the fraction of these producing secondary π-mesons of residual range R to be k_R. (We assume all τ-mesons in one plate to be concentrated at the centre of that plate; since the thickness of any one plate is about 3% of the total thickness of the stack, the error due to this assumption is never more than 1.5%).

The number of secondaries that originate in plate r which can be arrested is $k_R n_r f_{R,x_r}$. The total number of such secondaries stopped in the whole stack is $k_R \sum\limits_r n_r f_{R,x_r}$. The total number of these secondaries produced is $k_R \sum\limits_r n_r$ or $k_R N$, where N is the total number of τ-mesons found in the stack.

The total fraction of negative π-meson secondaries with residual range R, which are arrested in the stack is therefore given by:

$$F_R = \frac{\sum\limits_r n_r f_{R,x_r}}{N}\,.$$

APPENDIX II

DALITZ (6) has calculated on an empirical basis the probabilities of identifying the π^--meson in a stack of this size, for various E_{π^-} and $\cos\theta$ values. The results indicate that we expect to be unable to identify the π^--meson in less than 8% of the cases. This geometrical loss factor is greatest in the top right hand position of the Dalitz diagram and least at extreme values of E_{π^-}. The experimental histograms for all those τ-mesons in which the π^--meson was identified are shown in Fig. 8. The $\cos\theta$ distribution discriminates against all spin and parity combinations of the unique kind except those of $1+$ and $0-$. The large number of π^--mesons with energies less than 10 MeV discriminates against $1+$. The theoretical $0-$ curves, corrected for the above geometrical loss factor, are shown plotted in Fig. 8.

(6) R. H. DALITZ: Private communication, 1955.

RIASSUNTO (*)

Si sono eseguite misure sui mesoni π secondari originati dal decadimento dei mesoni τ positivi scoperti in un pacco di emulsioni nucleari esposte al fascio di mesoni K positivi del bevatrone di Berkeley. Si mostra che la distribuzione dell'energia di questi secondari è quella corrispondente all'ipotesi che il mesone τ sia una particella pseudoscalare e sembra escludere la possibilità che lo spin e la parità della particella siano 1^-, 1^+, 2^+ o 3^-.

(*) *Traduzione a cura della Redazione.*

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On a possible Scheme for Heavy Unstable Particles.

C. Ceolin and N. Dallaporta

Istituto di Fisica dell'Università - Padova
Istituto Nazionale di Fisica Nucleare - Sezione di Padova

(ricevuto il 10 Gennaio 1956)

Summary. — A level scheme is proposed for the heavy unstable particles and for their interactions with nuclei, in which an operator similar to a spin produces changes in «strangeness» analogous to the changes in charge produced by the isotopic spin. The proposed form of the interaction takes account of the experimental facts so far noted or admitted, the independent conservation of τ_3 and strangeness in strong interactions according to the ideas of Gell-Mann, and charge independence for interactions between nucleons and pions.

The fundamental idea underlying the different classifications that have been up to now proposed in order to explain the observed properties of heavy unstable particles consists essentially in the introduction of a new quantum number to which appropriate selection rules must apply ([1-5]). In the Gell-Mann formulation the following relation is assumed to hold for each particle between its charge C, its isotopic spin 3rd component τ_3, and its «strangeness» S

$$C = \tau_3 + S/2$$

(1) M. Gell-Mann and A. Pais: *Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics*, p. 342.

(2) R. G. Sachs: *Phys. Rev.*, **99**, 1573 (1955).

(3) A. Salam and J. C. Polkinghorne: *Nuovo Cimento*, **2**, 685 (1955).

(4) M. Goldhaber: *The Compound Hypothesis of the Heavy Unstable Particles*, II (preprint).

(5) A. Pais: *Proceedings of the Fifth Annual Rochester Conference*, 1955, p. 131.

apart from an additive constant which depends on the absolute definition of strangeness; the choice of Gell-Mann for it is different for fermions and bosons, and we shall not consider it here. Then the selection rule valid for strong interactions is that in each reaction between particles, total τ_3 and total S must be indipendently conserved.

If we accept for the baryon states only the hyperons definitely recognized up to now, plus Σ^0 which has some indirect evidence ([6]), and Ξ^0 which is as yet unknown but must be introduced in order to be able to apply isotopic spin formalism, we get the following well known Gell-Mann scheme for the baryons:

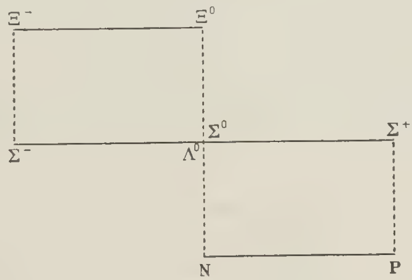


Fig. 1.

In what follows, we shall try to formulate some ideas which might be used to build a formalism which could allow the application to this scheme and to the phenomena related to heavy unstable particles of the theoretical concepts actually used to interpret normal nucleon-spin interaction.

Let us observe as a starting point, that in this scheme each row is labelled by a constant value of S, and each column by constant value of C. This seems to indicate a kind of symmetry between C and S and so we could think that if, generally speaking, C is expressed as a function of S by a relation of the kind:

(1) $$C = \alpha\tau_3 + \beta S \qquad (\alpha, \beta = \text{constants})$$

where τ_3 is the 3rd component of an angular momentum operator which distinguishes the different multiplet terms on the same row, we could equally well write from a formal point of view:

(2) $$S = \alpha'\omega_3 + \beta' C \qquad (\alpha', \beta' = \text{constants}),$$

where now ω_3 should be the 3rd component of another angular momentum

([6]) W. B. Fowler, R. P. Shutt, A. M. Thorndike and L. W. Whittemore: *Phys. Rev.*, 98, 121 (1955).

operator which might be thought to correspond to other rotations in 3-dimensional spin space which distinguishes the different multiplet terms on the same column. In other words, as different states of same strangeness and about same mass but different charge are considered as multiplet members of same isotopic spin, so we could consider different states of same charge but different mass and strangeness as multiplet members of this « ω spin ». In fact, a possibility of that kind has already been proposed and both the PAIS [5] and SALAM *et al.* [3] schemes consider relation (2) with the choice $\alpha' = 1$, $\beta' = 0$, and a special theoretical meaning for the operators τ and ω.

Now by considering relation (2) from a more general point of view, that is for different values of α, one can get different schemes, corresponding to different attributions of the ω_3 value to the different states. Or, to put it in the reverse way, if we should consider τ_3 and ω_3 as the typical quantum numbers of each state of particles, then C and S should be generally expressed through them by relations of the type:

$$C = A\tau_3 + B\omega_3 \tag{3}$$

$$S = A'\tau_3 + B'\omega_3 \tag{4}$$

and relations (1) and (2) should follow from (3) and (4) through elimination of ω_3 in the first and τ_3 in the second case; and coefficients $\alpha^{\alpha'}_{\Lambda}$, $\beta^{\beta'}_{\Lambda}$, should be related to A, B, A', B', by the relations:

$$\alpha = \frac{AB' - A'B}{B'}\,; \qquad \beta = \frac{B}{B'}\,,$$

$$\alpha' = \frac{AB' - A'B}{A}\,; \qquad \beta' = \frac{A'}{A}\,.$$

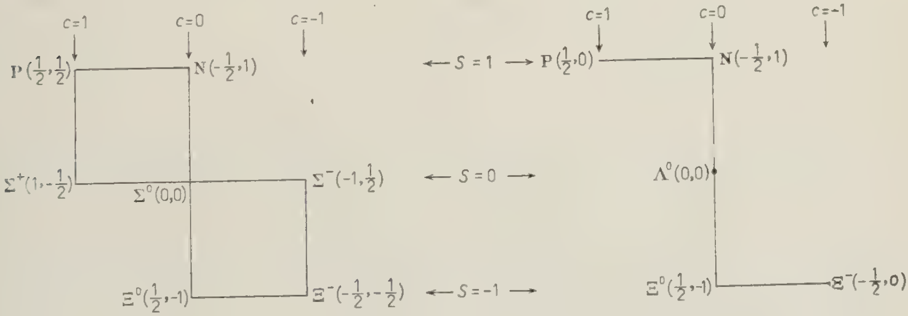
We get the Gell-Mann scheme in each case in which A, B, A', B', are choosen in such way that $\beta = \frac{1}{2}$ and $\alpha = 1$.

This introduction of the ω spin in a way absolutely symmetrical to the τ spin suggests on the other hand quite naturally an obvious extension of the formalism of ordinary nucleon-pion physics; in this field isotopic spin symbolism has proved particularly adequate in that the angular momentum properties of the three components are used to write in a very simple and condensed way the interaction terms which express the transformation of a given charge state into another charge state of the nucleon with absorption or emission of the meson. We can try to see if the same use can be made of the ω spin in order to express the transformation terms of nucleons into hyperons.

With this aim in view, we have tried to choose among the possibilities which are given by the apparent arbitrariness of choice of the four coefficients A, B, A', B', in formulas (3) (4) those cases which could be fit to interpret

correctly the known experimental data; in fact we have been able to find two such schemes, indicated in the following Fig. 2 by I and II, which combined together satisfy the preceding condition. The two numbers in brackets for each particle indicate the τ_3 and ω_3 value for it; while the common C and S-value are indicated for each column, viz for each row. Scheme I is obtained with the choice of coefficients ($A = B' = \frac{4}{3}$, $A' = B = \frac{2}{3}$, $\alpha = 1$, $\beta = \frac{1}{2}$); scheme II with ($A = B' = A' = 2$, $B = 1$, $\alpha = 1$ $\beta = \frac{1}{2}$). So both of them satisfy the Gell Mann relation.

In both cases, proton and neutron, and Ξ^0, Ξ^- are isotopic spin doublets, in scheme I however these doublets are connected only with the Σ hyperon triplet, in scheme II only with the Λ hyperon singlet. On the reverse, from the ω spin point of view, things are very different in the two cases: in scheme I P and Σ^+, Σ^- and Ξ^- are two ω spin Doublets while N, Σ^0 and Ξ^0 a ω spin triplet; in scheme II, P and Ξ^- are ω spin singlets, while N, Λ^0 and Ξ^0 constitute another ω spin triplet.



Scheme I Fig. 2. Scheme II

As we have eight different states corresponding to different combinations of values of charge and strangeness, we write the τ and ω of baryons as eight row square matrices, which when we label row and columns in the following order: P, N, Λ^0, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^- and which are given by:

For scheme I:

$$= \begin{vmatrix} \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} \end{vmatrix}; \tau_1 = \begin{vmatrix} \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -\sqrt{\frac{1}{2}} & \cdot & \sqrt{\frac{1}{2}} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot \end{vmatrix}; \tau_2 = \begin{vmatrix} \cdot & -i/2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ i/2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & i\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -i\sqrt{\frac{1}{2}} & \cdot & -i\sqrt{\frac{1}{2}} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & i\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -i/2 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & i/2 & \cdot \end{vmatrix}$$

$$\omega_{3_{\mathrm{I}}}=\begin{vmatrix} \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & -\frac{1}{2} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -\frac{1}{2} \end{vmatrix};\ \omega_{1_{\mathrm{I}}}=\begin{vmatrix} \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & -\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{1}{2} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{1}{2}} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} \\ \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \frac{1}{2} & \cdot & \cdot \end{vmatrix};\ \omega_{2_{\mathrm{I}}}=\begin{vmatrix} \cdot & \cdot & \cdot & -i/2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & i\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ i/2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -i\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot & \cdot & -i\sqrt{\frac{1}{2}} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -i/2 \\ \cdot & \cdot & \cdot & \cdot & i\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & i/2 & \cdot & \cdot \end{vmatrix}$$

For scheme II, the τ matrices are the same, while the ω matrices are:

$$\omega_{3_{\mathrm{II}}}=\begin{vmatrix} 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \end{vmatrix};\ \omega_{1_{\mathrm{II}}}=\begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot & \cdot & \sqrt{\frac{1}{2}} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix};\ \omega_{2_{\mathrm{II}}}=\begin{vmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & i\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & -i\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot & \cdot & -i\sqrt{\frac{1}{2}} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & i\sqrt{\frac{1}{2}} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}$$

It is easy to verify that these matrices obey to the commutation relations

$$\tau_\xi\tau_\eta-\tau_\eta\tau_\xi=i\tau_\varrho\,,\qquad \omega_\xi\omega_\eta-\omega_\eta\omega_\xi=i\omega_\varrho\,,$$

which are those of angular momenta. However generally the τ and ω matrices do not commute which each other.

We must now consider also how to represent the operators acting on mesons. According to Gell-Mann's ideas, K-particles (without distinction of the possible different types of K) have isotopic spin $\frac{1}{2}$.

If we now repeat for mesons what we have just said for baryons, and define for them the corresponding τ and ω operators, we are led for them to the following scheme analogous to Fig. 2,

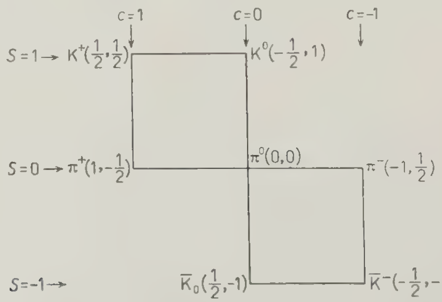


Fig. 3.

that is K^+, K^0 are an isotopic spin doublet, π^+, π^0, π^- a triplet and $\overline{K}^0$, $\overline{K}^-$ a doublet; while K^+, π^+ is an ω spin doublet, K^0, π^0, $\overline{K}^0$ a triplet and π^-, $\overline{K}^-$ a doublet; τ and ω operators are represented for them by square matrices of seven rows similar to those of scheme I for baryons, except that we can suppress the third row and column.

Let us now express by $\chi_{\alpha\beta}$ $(\alpha, \beta = 1, 2, 3)$ the creation and absorption operators of the K and π particles, in such a way that, if we put:

$$\chi_{++} = \frac{1}{2}(\chi_{11} - i\chi_{21} - i\chi_{12} - \chi_{22}) ; \qquad \chi_{+-} = \frac{1}{2}(\chi_{11} - i\chi_{21} + i\chi_{12} + \chi_{22}) ;$$

$$\chi_{-+} = \frac{1}{2}(\chi_{11} + i\chi_{21} - i\chi_{12} + \chi_{22}); \qquad \chi_{--} = \frac{1}{2}(\chi_{11} + i\chi_{21} + i\chi_{12} - \chi_{22}) ;$$

$$\chi_{3+} = \frac{1}{\sqrt{2}}(\chi_{31} - i\chi_{32}) ; \qquad \chi_{3-} = \frac{1}{\sqrt{2}}(\chi_{31} + i\chi_{32});$$

$$\chi_{+3} = \frac{1}{\sqrt{2}}(\chi_{13} - i\chi_{23}) ; \qquad \chi_{-3} = \frac{1}{\sqrt{2}}(\chi_{13} + i\chi_{23});$$

we may define:

$$\chi_{+-}|0\rangle = |\bar{K}^-\rangle; \quad \chi_{++}|K^+\rangle = |0\rangle; \quad \chi_{--}|0\rangle = |K^+\rangle; \quad \chi_{--}|\bar{K}^-\rangle = |0\rangle$$

$$\chi_{3+}|0\rangle = |K^0\rangle; \quad \chi_{3+}|K^0\rangle = |0\rangle; \quad \chi_{3-}|0\rangle = |K^0\rangle; \quad \chi_{3-}|\bar{K}^0\rangle = |0\rangle$$

$$\chi_{+2}|0\rangle = |\pi^-\rangle; \quad \chi_{+3}|\pi^+\rangle = |0\rangle; \quad \chi_{-3}|0\rangle = |\pi^+\rangle; \quad \chi_{-3}|\pi^-\rangle = |0\rangle$$

$$\chi_{33}|0\rangle = |\pi^0\rangle; \quad \chi_{33}|\pi^0\rangle = |0\rangle .$$

Let us further assume that:

$$\chi_{+-} = \chi_{-+} = 0$$

it then follows:

$$\chi_{11} = - \chi_{22} ; \qquad \chi_{21} = \chi_{12}$$

and

$$\chi_{--} = \chi_{11} + i\chi_{21} = - \chi_{22} + i\chi_{12} ; \qquad \chi_{++} = \chi_{11} - i\chi_{21} = - \chi_{22} - i\chi_{12} .$$

We may now easily write an hamiltonian for both creation and destruction of heavy unstable particles and ordinary interaction through pions. Let us however remark that scheme I will give us the possibility to excite only the Σ hyperonic states, and scheme II only the Λ one; therefore, if we want to obtain both possibilities we must use a linear combination of both schemes. In what proportion should both schemes enter in the interaction? The most

natural way of thinking consists in supposing that the probability for a nucleon to be excited in a Σ or Λ state will be proportional to the statistical factor, that is to the number of ways in which it will be able to reach a Σ or a Λ state. Now if we assume some model of the kind of the GOLDHABER (4) compound model for the hyperons, it is apparent that the number of ways for either a proton or a neutron to be excited according to scheme I is exactly double the number according to scheme II. Therefore we will give a weight 2 to scheme I as compared to II and try for a hamiltonian the following expression:

$$H=\bar{\psi}\Gamma S\psi \qquad \text{with} \qquad S=\sum_{1}^{3}{}_{\alpha\beta}\,[\tau_\alpha(\omega_{\beta_{\mathrm{I}}}\mp\tfrac{1}{2}\omega_{\beta_{\mathrm{II}}})+(\omega_{\beta_{\mathrm{I}}}\mp\tfrac{1}{2}\omega_{\beta_{\mathrm{II}}})\tau_\alpha]\chi_{\alpha\beta}\,.$$

The expression for Γ depends on the spin and parity of the mesons and could be different for different terms. We may verify that if we choose the minus sign in the hamiltonian we get charge independence for the nucleon-pion terms as ought to be. In fact, this choice of the coefficients in the hamiltonian is the only one which gives charge independence for nucleon pion terms, and it may be remarked that it has not been obtained by postulating it as such, but by another independent way.

It is very easy to verify that such an interaction hamiltonian gives rise only to terms which correspond exactly to reactions allowed by the associated production theory. For example let us apply it to P or N state. If we indicate the baryon wave function ψ by a column with 8 rows and indicate by η_{P}; η_{N}; η_{Λ^0}; η_{Σ^+}; η_{Σ^0}; η_{Σ^-}; η_{Ξ^0}; η_{Ξ^-}, the wave functions of P, N, Λ^0, Σ^+ and so on, and further define as usual $\tau_+=\tau_1+i\tau_2$; $\tau_-=\tau_1-i\tau_2$; $\omega_+=\omega_1+i\omega_2$; $\omega_-=\omega_1-i\omega_2$, we get:

$$S\eta_{\mathrm{P}}=-\sqrt{2}\eta_{\Sigma^0}\mathrm{K}^+ +\frac{1}{2\sqrt{2}}\eta_{\Lambda^0}\mathrm{K}^+ +\frac{3}{2\sqrt{2}}\eta_{\Sigma^+}\mathrm{K}^0+\frac{\sqrt{2}}{2}\eta_{\mathrm{N}}\pi^+ +\frac{1}{2}\eta_{\mathrm{P}}\pi^0\,,$$

$$S\eta_N=-\eta_{\Sigma^-}\mathrm{K}^+ +\frac{1}{2}\eta_{\Sigma^0}\mathrm{K}^0-\frac{1}{4}\eta_{\Lambda^0}\mathrm{K}^0+\frac{\sqrt{2}}{2}\eta_{\mathrm{P}}\pi^- -\frac{1}{2}\eta_{\mathrm{N}}\pi^0\,.$$

It is now easy to verify that total τ_3 commutes with the hamiltonian and is therefore a constant of motion. On the contrary, ω_3 does not commute, and this is quite natural: in fact some states such as the proton and Ξ^- possess different values of ω_3 in I and II scheme, so that at least for these cases, ω_3 does not possess a definite value. However both C and S operators commute, if we consider them defined through relations (3), (4) with different coefficients for schemes I and II which may be written as:

for scheme I

$$A=B'=\left|\begin{array}{cccccccc} \frac{4}{3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \frac{4}{3} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \frac{4}{3} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \frac{4}{3} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \frac{4}{3} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{4}{3} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \frac{4}{3} \end{array}\right|; \qquad A'=B=\frac{A}{2}$$

for scheme II

$$A'=A=B'=\left|\begin{array}{cccccccc} 2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 2 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 2 \end{array}\right|; \qquad B=\frac{A}{2}\,.$$

In this way the requisites of the Gell-Mann rules are verified and charge indipendence is also obtained but only for the nucleon pion terms. So the present scheme offers a formal possibility to account for the known experimental facts about elementary interactions, charge independence for nucleon-pion interaction and the Gell-Mann rules for the heavy unstable particles reactions; its simplicity comes from the fact that it starts directly from the experimental evidence concerning heavy unstable particles which suggests the two schemes I and II and from the application to them of the most natural extension of the isotopic spin operator.

This feature however is not disconnected from some rather more involved aspects of the formalism, the most prominent of which is the complicated relation between the two angular momenta, τ and ω. Both of them correspond to rotations in the isotopic spin 3 dimensional space: those however corresponding to the τ operations are such as to transform one into the other only states of the same configuration subspace, characterized by the same value of the strangeness; while those corresponding to the ω operations operate in the same way on states of same subspaces characterized by the same value of charge.

The complicated interconnection between these two kinds of subspaces which follows from the adopted experimental schemes is responsible for the non commutation of the τ with the ω operators and for the creation of a

relation between two angular momenta much more involved than is generally the case in similar problems in physics. One can however realize that it is perhaps somewhat artificially that each transformation between baryons has been decomposed and analyzed into two successive operations τ and ω and that really these two successive rotations are equivalent to a single operation equal to the product of the two ([7]). A weak point directly dependent on this very close connection of the two momenta τ and ω, is the extreme closeness of the scheme, that is the fact that the τ_3 and ω_3 numbers are so critically interconnected that the addition of any possible new hyperonic state which could be further discovered would almost surely spoil the scheme.

Finally we should point out that the automatic observation of the Gell-Mann rules in the interaction terms is not so much a virtue of the Hamiltonian than a consequence of having postulated for the meson tensor the condition $\chi{+-} = \chi{-+} = 0$.

* * *

We are indebted to Prof. L. RADICATI for some interesting discussions on this subject.

([7]) Since this work was completed, we came to know a paper by R. UTIYAMA: *Phys. Rev.*, **100**, 248 (1955) in which his line has been followed. Although his scheme is different from ours, the meaning of his $T_{\lambda\beta}$ operations is equivalent to a product of one τ and one ω operation of the present paper.

RIASSUNTO

Viene proposto uno schema per i livelli delle particelle pesanti instabili e per le interazioni di queste con i nucleoni nel quale un operatore simile ad uno spin produce i cambiamenti di stranezza in modo analogo ai cambiamenti di carica prodotti dallo spin isotopico. La forma di interazione proposta rende conto dei fatti finora sperimentalmente noti od ammessi, la conservazione indipendente di ξ_3 e della stranezza nelle reazioni forti secondo le idee di GELL-MANN e la indipendenza dalla carica per le sole reazioni tra nucleoni e pioni.

Un possibile schema generale di interazioni tra particelle elementari.

F. Duimio

Istituto di Scienze Fisiche dell'Università - Milano
Istituto Nazionale di Fisica Nucleare - Sezione di Milano

(ricevuto il 10 Gennaio 1956)

Riassunto. — Partendo da un'estensione del modello di Polkinghorne e Salam, atta a comprendere i leptoni, si danno prescrizioni, con le quali sembra possibile interpretare, almeno qualitativamente, i fenomeni di decadimento delle particelle elementari, sulla base di una interazione debole alla Fermi.

1. – Polkinghorne e Salam [1] hanno recentemente proposto uno schema per la classificazione di barioni e mesoni, fondato sull'ipotesi (introdotta da Pais [2,3]) che le varie particelle elementari corrispondano alle rappresentazioni del gruppo delle rotazioni in uno spazio euclideo a 4 dimensioni (spazio di carica). È noto che tali rappresentazioni possono essere contraddistinte da due «numeri quantici» τ' e μ' che possono essere assegnati a τ e μ, ciascuno generatore del gruppo delle rotazioni dello spazio a tre dimensioni, e connessi ai generatori $I_{\alpha\beta}$ del gruppo delle rotazioni nello spazio a 4 dimensioni dalle relazioni:

$$\left\{\begin{aligned} \tau_i &= \tfrac{1}{2}(I_{4i} + I_{jk})\,, \\ \mu_i &= \tfrac{1}{2}(I_{4i} - I_{jk})\,. \end{aligned}\right. \tag{1}$$

Lo schema di Polkinghorne e Salam si scosta da quello di Pais, in quanto

(1) A. Salam and J. C. Polkinghorne: *Nuovo Cimento*, 2, 685 (1955).
(2) A. Pais: *Proc. Nat. Acad. Sci.*, 40, 484 (1954).
(3) M. Gell-Mann and A. Pais: *Proc. Intern. Conference Glasgow*, 1954.

assume che le particelle elementari corrispondono solo a rappresentazioni tensoriali, cioè con $\tau' + \mu' =$ intero: la carica pertanto si ottiene sempre dalla relazione:

$$Q = I_{43} = \tau_3 + \mu_3 \,. \tag{2}$$

La classificazione proposta è quindi la seguente:

a) fermioni:

	Rappresent.	τ_3	μ_3
N^+, N^0	$(\frac{1}{2},\ \frac{1}{2})$	$\frac{1}{2}, -\frac{1}{2}$	$\frac{1}{2}$
Ξ^0, Ξ^-		$\frac{1}{2}, -\frac{1}{2}$	$-\frac{1}{2}$
Λ^0	$(0,\ 0)$	0	0
Σ^+, Σ^0, Σ^-	$(1,\ 0)$	$1, 0, -1$	0

b) bosoni:

π^+, π^0, π^-	$(1,\ 0)$	$1, 0, -1$	0
τ^+, τ^0, τ^-	$(0,\ 1)$	0	$0, 1, -1$
θ^+, θ^0	$(\frac{1}{2},\ \frac{1}{2})$	$\frac{1}{2}, -\frac{1}{2}$	$\frac{1}{2}$
$\bar{\theta}^0$, θ^-		$\frac{1}{2}, -\frac{1}{2}$	$-\frac{1}{2}$

Essa coincide con lo schema di Gell-Mann [3,4], salvo per l'introduzione del tripletto dei τ, accanto al doppietto delle θ (τ^0 dovrebbe essere altamente instabile).

2. – Le interazioni forti sono postulate essere quelle invarianti per rotazioni nello spazio di carica a 4 dimensioni.

Dato che le particelle sono contraddistinte dai valori τ' e μ' nei due spazi a tre dimensioni cui s'è accennato, ci sembra più opportuno usare una notazione che si riferisce direttamente a questi spazi, cioè:

N_{ij}	spinore semplice in ciascuno dei due spazi τ e μ
Λ^0	scalare in τ e μ
Σ_α	vettore in τ e scalare in μ
π_α	vettore in τ e scalare in μ
τ_α	scalare in τ e vettore in μ
θ_{ij}	spinore semplice in τ e μ.

(4) M. GELL-MANN: *Phys. Rev.*, **92**, 833 (1954).

Le interazioni, invarianti in ciascuno dei due spazi, saranno del tipo:

$$(3)\qquad \begin{cases} \bar{N}_{ij}\sigma^{\alpha}_{ih}\delta_{jk}N_{hk}\pi_{\alpha} \\ \bar{N}_{ij}\delta_{ih}\sigma^{\alpha}_{jk}N_{hk}\tau_{\alpha} \\ \bar{N}_{ij}\delta_{ih}\delta_{jk}\Lambda^{0}\theta_{hk} \\ \bar{N}_{ij}\sigma^{\alpha}_{ih}\delta_{hk}\Sigma_{\alpha}\theta_{hk} \end{cases}$$

con

$$N_{11} = \Xi^{-} \qquad N_{21} = \Xi^{0}$$

$$N_{12} = \mathrm{N}^{0} \qquad N_{22} = \mathrm{N}^{+}$$

e analoghe per le θ, mentre per le Σ_{α} (e le τ_{α}) si hanno relazioni analoghe a quelle per i π (*)

$$\Sigma = 1/\sqrt{2}(\Sigma_1 + i\Sigma_2)$$

$$\Sigma^{*} = 1/\sqrt{2}(\Sigma_1 - i\Sigma_2)$$

$$\Sigma^{0} = \Sigma_3 .$$

Con queste notazioni è facile scrivere un'interazione invariante per rotazioni intorno all'asse 3 dello spazio τ o dello spazio μ, con un campo scalare negli spazi di carica, ad esempio,

$$(4)\qquad \bar{N}_{ij}\{\sigma^{3}_{ih}\delta_{hk} + \delta_{ih}\sigma^{3}_{hk}\}N_{hk}\cdot A .$$

È facile constatare che se A è il campo elettromagnetico la (4) dà nel caso considerato la forma corretta dell'interazione elettromagnetica.

3. – Oltre alle interazioni sopra accennate, forti ed elettromagnetiche, devono esistere anche interazioni deboli, che rendono conto delle lunghe vite medie di particelle come le Λ^0, θ, π, ecc.

D'altro canto, per avere un quadro generale, bisogna allargare lo schema a comprendere i fermioni leggeri (elettroni, neutrini e μ). Le interazioni di questi ultimi tra di loro e con le altre particelle è noto che devono essere deboli (ed elettromagnetiche naturalmente).

Introduciamo ora le seguenti ipotesi:

1) L'insieme delle particelle e^-, ν, e μ^+, μ^0 costituiscono una rappresentazione $(\frac{1}{2}, \frac{1}{2})$ del gruppo delle rotazioni nello spazio a 4, analogamente alle particelle N e Ξ. Useremo per esse la notazione n_{ij}.

(*) Essendo i Σ fermioni, è chiaro che volendo scrivere dettagliatamente le interazioni (3), bisognerà tenere conto della differenza dell'operazione di coniugazione di carica per bosoni e fermioni.

2) La differenza tra le «famiglie» di particelle N_{ij} ed n_{ij} consiste nel fatto che la prima è supposta possedere «carica mesonica» 1 (+1 per le particelle e — 1 per le antiparticelle), intendendo con esse la capacità di interagire coi campi mesonici (π, τ e θ) e la seconda ha «carica mesonica» uguale a zero (ciò equivale a dire che non esistono interazioni dirette leptone-mesone). È chiaro come da tale ipotesi segue la legge di conservazione dei barioni (5).

3) Tutte le interazioni deboli hanno la forma di una interazione universale alla Fermi tra quattro fermioni qualunque. Naturalmente esse dovranno essere tali da conservare la carica totale e la «carica mesonica» totale. Potremo cioè indicarle in generale con

$$g_F(\bar{A}B)(\bar{C}D)\cdot\delta\,(\text{carica elettrica})\cdot\delta\,(\text{carica mesonica})\,. \tag{5}$$

Inoltre, generalizzando un'ipotesi formulata da KONOPINSKI e MAHMOUD (6), supporremo che

4) Nell'interazione (5), i campi A, B, C, D descrivono sempre particelle (non antiparticelle), cioè ogni processo possibile è descrivibile con la distruzione di due particelle «normali» e la creazione di due altre:

$$A + C \to B + D\,.$$

In tal modo processi come, ad esempio,

$$\mu^+ \to e^+ + e^- + e^+ \tag{6}$$

(distruzione di 3 particelle (*) e creazione di una) sono proibiti, in accordo con l'esperienza.

Con queste ipotesi tutti i decadimenti delle particelle instabili finora note possono essere interpretati come procedenti attraverso un'interazione (5).

4. – L'insieme delle ipotesi sopra esposte costituisce uno schema coerente, in base al quale è possibile rendere conto degli aspetti essenziali di tutti i fenomeni di interazione e di decadimento delle particelle elementari, finora noti.

Esso può essere considerato come una riformulazione definita ed univoca, basata sul modello di POLKINGHORNE e SALAM, di considerazioni già sviluppate

(5) Considerazioni di questo tipo sono state sviluppate in S. ONEDA: *Prog. Theor. Phys.*, **9**, 327 (1953); S. ONEDA and H. UMEZAWA: *Prog. Theor. Phys.*, **9**, 685 (1953).

(6) E. J. KONOPINSKI and H. M. MAHMOUD: *Phys. Rev.*, **92**, 1045 (1953).

(*) È chiaro che se i leptoni sono raggruppati nella famiglia n_{ij}, μ^+ è da considerarsi particella normale e μ^- antiparticella, in accordo con l'ipotesi avanzata in cit. (6).

da DALLAPORTA e GELL-MANN [7]; e conferma essenzialmente la possibilità che tre tipi di interazioni fondamentali: *a*) forte, mesone-barione; *b*) intermedia, elettromagnetica; *c*) debole, universale alla Fermi, siano sufficienti a rendere conto dei fenomeni osservati nel campo delle particelle elementari (*).

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[7] N. DALLAPORTA: *Nuovo Cimento*, **1**, 962 (1955); M. GELL-MANN: *Intern. Phys. Conference Pisa*, 1955 (in corso di stampa sul *Suppl. Nuovo Cimento*); F. DUIMIO: *Nuovo Cimento*, **2**, 1308 (1955).

(*) Il prof. KROLL, in una utile discussione, ha fatto notare come con le ipotesi sopra esposte, i processi

$$\Xi^- \to \Lambda^0 + \pi^-$$

$$\Xi^- \to n + \pi^-$$

sarebbero entrambi permessi, mentre solo il primo è osservato sperimentalmente.

Solo un accurato esame dei due suddetti processi, in un certo senso analogo a quello eseguito da vari autori sulla forma dell'interazione di Fermi responsabile del decadimento dei mesoni π in leptoni, potrà dire se anche il decadimento della particella Ξ^- è inquadrabile nello schema proposto in questa nota.

Vivi ringraziamenti vanno pertanto al prof. KROLL e al prof. CALDIROLA per il loro gentile interessamento.

SUMMARY

Starting from an extension of Polkinghorne-Salam's model, in which also leptons find place, one gives some prescriptions by which it seems possible to explain qualitatively the decay phenomena of the elementary particles, on the basis of a weak Fermi interaction.

The Capture of Negative Hyperons.

S. J. Goldsack and W. O. Lock
Department of Physics, University of Birmingham

(ricevuto il 12 Gennaio 1956)

Summary. — Three events are described which probably represent the capture of negative hyperons. In each case a single fast proton is emitted of energy between 60 and 70 MeV. The present experimental data on Y^- capture are summarized and briefly discussed.

1. – Introduction.

Ilford G5 emulsions exposed to the 5.7 GeV proton beam from the Berkeley Bevatron have been searched at Birmingham for stars containing « hammer » tracks. In the course of this work two events were found which appear to represent the capture of negative hyperons. A similar event had previously been found at Berkeley by G. Goldhaber and one of us (S.J.G.) (*). Because of the striking similarity between these events it appeared to us to be of interest to report them and to consider our present knowledge of negative hyperon events.

2. – Experimental Observations.

Details of the events to be described are given in Table I. The primary tracks are all short: Bm_2 and Bk_1 are so short that practically no measurements are possible. Bm_1 is 105 μm long and quite flat. We have applied the constant sagitta scattering technique Dilworth, Goldsack and Hirschberg (1) ex-

(*) We are grateful to Dr. Goldhaber for sending us the details of this event.
(1) C. C. Dilworth, S. J. Goldsack and L. Hirschberg: *Nuovo Cimento*, **11**, 113 (1954).

TABLE I.

Event	Primary		Secondaries		
	Parent star	Length in mm	Length in mm	No. of emulsions	Energy MeV (assumed proton)
Bm_1	18+3p	0.105	(*a*) 5.0 (o) (*b*) blob 1 μ	5	65 ± 10
Bm_2	23+3p	0.009	(*a*) 13.2 (t) (*b*) blob	5	61 ± 5
Bk_1	12+1p	0.028	(*a*) 12.0 (o) (*b*) blob	1	70 ± 10

(o) = observed range; (t) = total range.

tended to shorter ranges, to it and obtain a scattering $\bar{D}$ of $0.16 \pm .03$ μm per «π-cell». Nine π-mesons measured over the last 100 μm of their range by the same technique gave a mean $\bar{D}$ of $0.40 \pm .03$ μm per «π-cell». Thus the mass of the particle Bm_1 is $2300\,^{+1700}_{-\,700}\,m_e$. This error would allow the particle to be a K-meson but makes it unlikely that it is a π^-.

In each of these events the emitted particle comes to rest and from the end point of its track there comes a single high energy proton and nothing else, except that in each case there is a tiny blob suggestive of a recoil, which would probably be classed as uncertain in any one case.

The secondary of Bm_1 has a grain density 55 ± 5 grains per hundred μm and passes through five emulsions before leaving the stack. The observed range is 5 mm. The estimated total range, obtained from measurements of grain density and scattering, is 1.5 ± 0.4 cm, equivalent to an energy (if a proton) of 65 ± 10 MeV.

The secondary of Bm_2 comes to rest in the stack after a path of 13.2 mm in the emulsion; from grain density and range it is identified as a proton of energy 61 ± 5 MeV.

The secondary of Bk_1 has a relative grain density of $g/g_0 = 3.9 \pm 0.2$ and travels in one emulsion for 12 mm before leaving the stack. The grain counts at the two ends are consistent with a proton of 70 ± 10 MeV.

The striking similarity between these three events led us to look for a common explanation. The energy of the secondary protons indicate that they are not due to known K^+ or Y^+ decays, and the «blobs» also confirm this. The very low energy of the emitted particles suggests a negative charge (cf. π^- and π^+ ejection). The particles are all emitted from heavy nucleus interactions which seldom lead to α-particles or protons of less than 50 μm range because of the coulomb barrier effect.

Since we rule out the possibility that Bm_1 is a π^- particle, the simplest common explanation of all three events seems to be that they represent the nuclear capture of negative K-particles or negative hyperons. The mass value for Bm_1 suggests the latter possibility. Of course both Bm_2 and Bk_1 could be π^- particles but the extensive data on π^- capture stars shows that the emission of a $60 \div 70$ MeV proton is rather infrequent. Neither can we exclude the possibility that we are dealing with (for example) a deuteron or triton hyperfragment which decays non-mesonically. In what follows we assume that the three events represent the decay of negative hyperons but we consider the experimental data with and without their inclusion.

3. – Discussion.

3'1. *Theoretical.* – FRIEDLANDER *et al.* [2] have pointed out that on the basis of the Gell-Mann–Nishijima models, the most probable Y^- capture process is

$$Y^- + p \to \Lambda^0 + n + Q\,,$$

where $Q = 78$ MeV. If both the neutron and the Λ^0 escape the available excitation will be of the order of 20—30 MeV, and the residual nucleus will have a neutron excess of one. Such a process will often lead to no visible interaction. If the neutron or Λ^0 is retained inside the nucleus the excitation energy is greater (up to 78 MeV if both are captured), but the neutron excess is now two, or three if we count the Λ^0. It seems likely that this part of the process will frequently lead to no visible star. This point has also been made by FRY, SCHNEPS, SNOW and SWAMI [3] who estimate that about 50% of Y^- stars give no visible prong.

If the Λ^0 is captured the residual nucleus may be considered as equivalent to an excited hyperfragment of rather high charge. We may compare the type of disintegration observed in Y^- capture with that observed for hyperfragments emitted from stars.

The decay of hyperfragments may take place either with or without the emission of a π-meson, that is, either the Λ^0 decays according to its normal scheme

$$\Lambda^0 \to p + \pi^- + 37\ \text{MeV} \tag{1}$$

(2) M. W. FRIEDLANDER, Y. FUJIMOTO, D. KEEFE and M. G. K. MENON: *Nuovo Cimento*, **2**, 90 (1955).

(3) W. F. FRY, J. SCHNEPS, G. A. SNOW and M. S. SWAMI: *Phys. Rev.*, **100**, 950 (1955).

or it interacts with a nucleon e.g.

$$\Lambda^0 + N \to n + n + 180\ \text{MeV}\,. \tag{2}$$

The mesonic decay is favoured for fragments of low Z. For fragments of high Z, as shown by CHESTON and PRIMAKOFF [4] and observed experimentally [5], the non-mesonic decay is much more probable. For a Λ^0 captured in a nucleus of the emulsion π^- emission should therefore be very rare.

3·2. *Experimental.* – To our knowledge there are 17 possible cases of Y^- capture (*). Twelve are summarized in Table I of the paper by CECCARELLI *et al.* [6]. The extra cases, other than the three possible events reported in this paper, are given in Table II.

TABLE II.

Event	Primary		Secondary Star
	Origin	Length in mm	
GeMi$_4$ [7]	18+3p	0.260	30 MeV π + 15 μm recoil
Chicago [8]	9+0π	12.62	${}^1_4$H* fragment + 2 short recoils

It is striking that in the three cases of π-meson emission the energy of the meson is in each case about 30 MeV. Further in each case the primary particle is too short for accurate mass determinations to be made. It was pointed out by LEVI-SETTI at the Pisa Conference that the event GeMi$_4$ could in fact be a ${}^4_1$H hyperfragment decaying according to the scheme

$${}^4_1\text{H}^* \to {}^3_2\text{He} + n + \pi^- + Q\,.$$

The Bombay event Bo$_1$ has two short recoil tracks (6 μm and 16 μm) in addition to the π-particle. This could also be the disintegration of ${}^4_1$H* according to one of the schemes

$$\begin{aligned}{}^4_1\text{H}^* &\to {}^2_1\text{H} + p + n + \pi^- + Q\\ &\to p + p + n + n + \pi^- + Q\,.\end{aligned}$$

(4) W. B. CHESTON and H. PRIMAKOFF: *Phys. Rev.*, **92**, 1537 (1953).
(5) C. CASTAGNOLI, G. CORTINI and C. FRANZINETTI: *Nuovo Cimento*, **2**, 550 (1955).
(6) C. CECCARELLI, N. DALLAPORTA, M. GRILLI, M. MERLIN, G. SALANDIN, B. SECHI and M. LADU: *Nuovo Cimento*, **2**, 542 (1955).
(*) We exclude the 12 cases mentioned by FRY *et al.* (3) since no details of these events (which came from K⁻ capture stars) have yet been reported.
(7) L. BACCHELLA, M. DI CORATO, M. LADU, R. LEVI-SETTI and L. SCARSI: *Preliminary Report of Pisa Conference* (1955), p. 299.
(8) M. SCHEIN, D. M. HASKIN and D. LEENOV: *Preprint September* (1955).

This was pointed out to us by FRIEDLANDER and KEEFE (Private communication) who have also re-analysed the Bristol event Y-Br_{12} and now believe it to be a further case of the disintegration of ${}^4_1H^*$.

Thus the evidence for any cases of π-meson emission following the capture of a Y^- particle is very slight. It seems most probable that when the Λ^0 does not escape from a nucleus it undergoes non-mesonic decay according to (2). During this process a total of 180 MeV is released. At first sight one might expect this to lead to a star similar in characteristics to that of a π^- capture star, which releases 140 MeV. However, the distribution of the energy among the emitted particles may well be different. That indeed it is so is shown in Table III which gives the prong distribution for π^- capture stars (9) and for

TABLE III.

Events	No. of prongs							
	1	2	3	4	5	6	7	8
π^- (%)	34	31	22	11	2	—	—	—
Y^-	1	3	2	—	1	1	1	—

the probable Y^- events (*excluding* the events reported in this paper and the two cases in which a hyperfragment is ejected from a Y^- capture). In this table a prong is defined as a recognisable track of any length since the π^- data are given in this way. If we consider as prongs only those tracks with a length greater than 5 μm (as is used in the study of nuclear disintegrations) we obtain the prong distribution given in Table IV.

TABLE IV.

	No. of prongs > 5 μm range							
	1	2	3	4	5	6	7	8
Y^-	5	1	—	1	1	—	1	—
Visible energy in MeV	9, 17, 25, 65, 100	140	—	35	20	—	110	—

The last line of this table gives the observed visible energy of the stars; we have counted the energy of a recoil as 1 MeV per micron. The stars appear to group into those of large visible energy and those of small visible energy. It was pointed out by CECCARELLI *et al* (6) that stars with less than 50 MeV visible energy are probably examples of interaction in which the Λ^0 escaped

(9) M. G. K. MENON, H. MUIRHEAD and O. ROCHAT: *Phil. Mag.*, **41**, 583 (1950).

while higher energy stars represent the events in which the Λ^0 was captured. Thus three one prong and the four and five prongs events all correspond to the first process, the remainder to the second.

Within the very meagre data on the high energy group there appears be to a tendency for a single high energy proton to be emitted. This is especially true if the present events are included.

A partial explanation of this effect may be found in the details of the interaction. A π^- particle is normally captured by two nucleons. The chance of its capture by a proton and a neutron is considerably greater than that for two protons, because of the Pauli principle. Thus the most common process for π^- is

$$\pi^- + p + n \to n + n + 140\ \text{MeV}\ .$$

For the case of a Λ^0 either a proton or a neutron alone will be sufficient for the disintegration.

Thus the processes

$$\Lambda^0 + p \to p + n + 180\ \text{MeV}$$

$$\Lambda^0 + n \to n + n + 180\ \text{MeV}$$

should be approximately equally frequent. The nucleons are emitted with about 90 MeV each, and so the presence of some high energy protons is to be expected. The energy spectrum of the protons from hyperfragments does in fact show a significant number with energy above 50 MeV. (The data are summarized by CASTAGNOLI *et al.* in Fig. 10 of their recent paper ([5])).

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We wish to express our thanks to Dr. M. W. FRIEDLANDER and Dr. D. KEEFE for their comments on the first draft of this paper; to Dr. R. H. DALITZ and Dr. P. T. MATTHEWS of the Mathematical Physics Department for many informative discussions; to Mr. B. A. MUNIR who found the event Bm_1 and helped with the measurements; to Miss M. J. LEWIS who found event Bm_2. We are especially grateful and to Dr. E. J. LOFGREN and his associates in the Bevatron team at Berkeley for kindly exposing an emulsion stack to the 5.7 GeV proton beam.

RIASSUNTO (*)

Si descrivono tre eventi che probabilmente rappresentano catture di iperoni negativi. In ognuno dei casi si ha emissione di un protone veloce singolo di energia compresa tra 60 e 70 MeV. Si riassumono, discutendoli brevemente, i dati sperimentali attualmente disponibili sulla cattura degli Y^-.

(*) *Traduzione a cura della Redazione.*

On Generalised Dispersion Relations II.

ABDUS SALAM
St. John's College - Cambridge

W. GILBERT
Trinity College - Cambridge

(ricevuto il 16 Gennaio 1956)

Summary. — Considerations of an earlier paper are generalised to write down non-forward scattering, spin-flip dispersion relations. The contribution from the « bound state » is discussed.

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1. – This paper continues the discussion of an earlier paper [1]. A meson of 4-momentum k $(k^2 = \mu^2)$ is scattered by a nucleon of initial 4-momentum p, $(p^2 = \varkappa^2)$ spin λ; no other particles are emitted in the finl state, the meson and the nucleon momenta being k' and p', and spin λ'. From energy conservation,

$$k' = p + k - p' \, , \qquad k\cdot(p - p') = p\cdot p' - \varkappa^2 \, . \tag{1}$$

In the earlier paper the scattering amplitude M_R defined covariantly, was written as

$$M_R(\lambda'\lambda) = \bar{u}^{\lambda'}(p')\{L' + i\gamma k M'\}u^{\lambda}(p) \, , \tag{2}$$

where L' and M' depend on $k\cdot(p+p')$ and $p\cdot p'$.

To obtain dispersion relations it was found necessary to work in a special

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(1) A. SALAM: *Nuovo Cimento*, **3**, 424 (1956).

(2) Notation: $p = \boldsymbol{p}$, $p_4 = ip_0$, $\gamma_i = \begin{pmatrix} & \sigma_i \\ \sigma_i & \end{pmatrix}$, $\gamma_4 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$. Define $i\gamma p = i\boldsymbol{\gamma}\cdot\boldsymbol{p} + i\gamma_4 p_4$; $p\cdot q = p_0 q_0 - \boldsymbol{p}\cdot\boldsymbol{q}$. Thus $(i\gamma p)^2 = p^2$.

frame of reference defined by $\boldsymbol{p}+\boldsymbol{p}'=0$. If $\boldsymbol{p}=0, 0, P$, $\boldsymbol{k}=Q, 0, -P$ $p\cdot p'-\varkappa^2=2P^2$, and $k\cdot(p+p')=2k_0p_0$. Further (3) in this frame

$$\text{(3)} \qquad \begin{cases} u^1(p')\{1, i\gamma k\}u^1(p) = \dfrac{p_0}{\varkappa}, k_0 \\[2ex] u^2(p')\{1, i\gamma k\}u^1(p) = 0, \dfrac{PQ}{\varkappa}. \end{cases}$$

In reference (1), a dispersion relation was obtained for $M_R(11)=$ $=(p_0/\varkappa)L'+k_0M'$. It is in fact equally simple to obtain relations for L' and M' separately.

Consider $M_R(21)$, the spin-flip amplitude in this frame. $M_R(21)=-M_R(12)=$ $=(PQ/\varkappa)M'$. Now

$$\text{(4)} \qquad (p'2|[j(0), j(x_0, -x_1, x_2, x_3)]|p1) = \quad (p'1|[j(0)j(x)]|p2)$$

$$\text{(5)} \qquad (p'2|[j(0), j(x_0, x_1, x_2, -x_3)]|p1) = -(p2|[j(0)j(x)]|p'2)\,.$$

These results follow by using the parity operator; for example

$$P_{x_3}j(x)P_{x_3}^{-1} = -j(x_0, x_1, x_2, -x_3)\,; \qquad P_{x_3}|p_0, \boldsymbol{p}, 1\rangle = i|p_0, -\boldsymbol{p}, 1\rangle\,.$$

From $M_R(12)=-M_R(21)$, and using (4) and (5),

$$M_R(21) = -\tfrac{1}{2}\int\theta(-x)\exp[-ik_0x_0]\sin Qx_1\{J_1(P, x)\sin Px_3 + J_2(P, x)\cos Px_3\}\,\mathrm{d}^4x.$$

Here

$$J_1(x) = \{(p'2|p1)-(p1|p'2)\} - \{(p'1|p2)-(p2|p'1)\}$$

$$J_2(x) = i\{(p'2|p1)+(p1|p'2)\} - i\{(p'1|p2)+(p2|p'1)\}$$

in an obvious notation. J_1, J_2 are real. From the contour (4) integral

$$\int\frac{\mathrm{d}k_0'\exp[-ik_0'x_0]}{k_0'^2-k_0^2}\left(\frac{\sin Q'x_1}{Q'}\right),$$

(3) The (positive energy) free particle spinors are defined as

$$u_\alpha^\lambda(p) = \frac{1}{N(p)}(i\gamma p - k)_{\alpha\lambda}\,, \qquad \lambda = 1, 2\,; \qquad \alpha = 1, 2, 3, 4\,.$$

We obtain $N(p)$, the « invariant » normalization from $u^{\lambda'}(p')u^\lambda(p)=\delta_{\lambda'\lambda}$. Thus $N^{-2}(p)=2\varkappa(\varkappa+p_0)$.

(4) M. L. GOLDBERGER: *Phys. Rev.*, **99**, 979 (1955).

we infer

$$\frac{\operatorname{Re} M'(k_0, P)}{k_0} = \frac{2\text{ p.v.}}{\pi} \int_0^\infty \frac{\operatorname{Im} M'(k_0', P)\,\mathrm{d}k_0'}{k_0'^2 - k_0^2}\,. \tag{6}$$

Clearly $k_0 M'(k_0, P)$ would satisfy a dispersion relation as in eq. (27) of reference [1]. Thus also

$$\operatorname{Re} L'(k_0, P) = \frac{2\text{ p.v.}}{\pi} \int_0^\infty \frac{\operatorname{Im} L'(k_0', P) k_0'\,\mathrm{d}k_0'}{k_0'^2 - k_0^2}\,. \tag{7}$$

L'_R and M'_R are functions of invariant scalar products $k\cdot(p+p')$ and $p\cdot p'$. Relations (6) and (7) can thus be expressed entirely in terms of invariant quantities.

2. – It is perhaps useful to set down the transformation formulae from the frame used, to the conventional centre of mass frame, defined from the requirement $\boldsymbol{k}^c + \boldsymbol{p}^c = 0$. The superscript «$c$» distinguishes c.m. quantities.

Let $p^c_\mu = a_{\mu\nu} p_\nu$. The matrix $(a_{\mu\nu})$ is given by

$$a_{11} = a_{44} = \cosh\chi\,, \qquad a_{14} = -a_{41} = i\sinh\chi\,, \qquad a_{22} = a_{33} = 1$$

where $\operatorname{tg} h\chi = Q/(p_0 + k_0)$.

Define

$$S\gamma_\nu S^{-1} = a_{\mu\nu}\gamma_\mu\,.$$

For the $a_{\mu\nu}$ in (4),

$$S = \exp\left[(i/2)\,\gamma_1\gamma_4\chi\right] = c + i\gamma_1\gamma_4 s\,.$$

Here

$$c = \cosh\chi/2\,, \qquad s = \sinh\chi/2\,.$$

If $u^{(s\lambda)}(p^c) = Su^\lambda(p)$, clearly

$$\bar{u}^{\lambda'}(p')\{1, i\gamma k\}u^\lambda(p) = u^{(s\lambda')}(p'^c)\{1, i\gamma k^c\}u^{(s\lambda)}(p^c)\,. \tag{8}$$

Now

$$Su^\lambda(p)|_\alpha = \frac{1}{N(p)}\,S(i\gamma p - \varkappa)|_{\alpha\lambda} = \frac{1}{N(p)}\,(i\gamma p^c - \varkappa)S|_{\alpha\lambda}\,. \tag{9}$$

Considered as a 4×4 matrix, only two of the columns of $(i\gamma p^c - \varkappa)$ are

linearly independent. Therefore

$$(10)\qquad Su^1(p) = [(p_0 + \varkappa)(p_0^c + \varkappa)]^{-\frac{1}{2}}\{c(p_0 + \varkappa)u^1(p^c) - Ps\,u^2(p^c)\}$$

$$(11)\qquad Su^2(p) = [(p_0 + \varkappa)(p_0^c + \varkappa)]^{-\frac{1}{2}}\{Ps\,u^1(p^c) + c(p_0 + \varkappa)\,u^2(p^c)\}.$$

Define

$$(12)\qquad M_R^c(11) = \bar{u}^1(p^c)\{L'_R + i\gamma k^c M'_R\}\,u^1(p^c)\,, \quad \text{etc.}$$

From (10) and (11),

$$(13)\qquad M(11) = \alpha M^c(11) + \beta M^c(21) \ \text{[5]}$$

$$(14)\qquad M(21) = -\beta M^c(11) + \alpha M^c(21)$$

where

$$\beta = \frac{P \sinh \chi}{p_0^c + \varkappa}\,, \qquad \alpha^2 + \beta^2 = 1\,.$$

$M^c(11)$ corresponds to no spin-flip, and $M^c(21)$ to the spin-flip amplitudes in the c.m. system (the latter with no azimuth change). In terms of the phase shifts, therefore

$$(15)\qquad M^c(11) = \left(\frac{4\pi}{\varkappa}\right)\frac{p_0^c + k_0^c}{k_c}\sum\,(la_{l-} + (l+1)a_{l+})P_l(\cos\theta_c)\,,$$

$$(16)\qquad M^c(21) = \left(\frac{4\pi}{\varkappa}\right)\frac{p_0^c + k_0^c}{k_c}\sum\,(a_{l-} - a_{l+})\sin\theta_c\,\frac{\mathrm{d}}{\mathrm{d}(\cos\theta_c)}\,P_l\,(\cos\theta_c)\,.$$

Here $a = e^{2i\delta} - 1/2i$, and δ_{l-} and δ_{l+} refer to phase shifts, corresponding to $j = l - \frac{1}{2}$, and $j = l + \frac{1}{2}$. k_c and θ_c are the 3-momentum and the angle in the c.m. system. To obtain k_c and θ_c, note $p\cdot p' - \varkappa^2 = k_c^2(1 - \cos\theta_c) = 2P^2$, and $k\cdot p = k_0 p_0 + P^2 = k^c p_0^c + k_c^2$.

3. – The case of charged mesons has been completely solved by GOLDBERGER [4]. Write

$$M_{R\alpha\beta} = \delta_{\alpha\beta}\,\bar{u}(L' + i\gamma k M')u + \tfrac{1}{2}[\tau_\alpha, \tau_\beta]\,\bar{u}(L'' + i\gamma k M'')u\,.$$

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[5] On account of our choice of axes ($\boldsymbol{p} = 0, 0, P$; $\boldsymbol{k} = Q, 0, -P$), $M(11) = M(22)$ and $M(12) = -M(21)$. The same holds for the trasformed quantities M^c.

Then L' and M'' satisfy a relation of the type (6); M', L'' a relation of the type (7). L' and M' refer to the phase shift combination $\frac{1}{3}a(\frac{1}{2}) + \frac{2}{3}a(\frac{3}{2})$, and L'' and M'' to $a(\frac{1}{2}) - a(\frac{3}{2})$.

In relations (6) and (7), $k_0 \geqslant (\mu^2 + P^2)^{\frac{1}{2}}$ is the physical range; $(\varkappa\mu - P^2)/2p_0 \leqslant k_0 < (\mu^2 + P^2)^{\frac{1}{2}}$ is the « unphysical range ». Below $(\varkappa\mu - P^2)/2p_0$, the so called « bound state » contributes to $\operatorname{Im} M_R$. The exact energy dependence of this term can be computed if we adopt the convention,

$$\Gamma_5(p_1^2 = \varkappa^2,\ p_2^2 = \varkappa^2,\ (p_1 - p_2)^2 = \mu^2) = gZ^{-1}\gamma_5 = g_1\gamma_5\,, \quad (i\gamma p_1 + \varkappa = i\gamma p_2 + \varkappa = 0),$$

for the renormalization of the coupling constant in meson theory. The « bound state » contribution then involves the (unknown) renormalized coupling constant g_1, and has the form,

$$\operatorname{Im} M_{R\alpha\beta}(\lambda'\lambda) = -\left(\frac{g_1^2}{4\pi}\right)\cdot 4\pi^2\tau_\alpha\tau_\beta\left\{\bar{u}^{\lambda'}(i\gamma k)u^{\lambda}\right\}\delta(\mu^2 + 2P^2 - 2k_0p_0)\,.$$

The renormalized coupling constant defined from $\Gamma_5(p_1 = p_2) = g_1'\gamma_5$ then differs from the above by $O(\mu^2/\varkappa^2)$.

After the work described here was completed, the authors, learnt from Prof. M. L. GOLDBERGER that he has obtained identical results in collaboration with Professors NAMBU and OEHME, and is applying the relations to calculate S phase shifts.

RIASSUNTO (*)

Si generalizzano le considerazioni di un precedente lavoro per scrivere le relazioni dello scattering non in avanti, e della dispersione per inversione di spin. Si discute il contributo dello « stato legato ».

(*) *Traduzione a cura della Redazione.*

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A Mass Spectrum from a Field Theory Model of the Non-Local Theory.

P. Sen (*)

Dehra Dun - India

(ricevuto il 17 Gennaio 1956)

Summary. — A field theory model of Yukawa's non-local theory for the mass spectrum of the elementary particles is built. Corresponding to every spin a wave equation is found whose ground state represents the mass of the stable particle of that spin and its excited states are interpreted as the elementary unstable particles. Allowing for the choice of suitable values for the arbitrary coupling constants, the theory appears to be in reasonable agreement with the general features of the mass spectrum and gives the mass 2247 m_e for $\Lambda^{+,0,-}$ particles of spin $\frac{1}{2}$. Particles of spin 0 and masses 1640 m_e, 2287 m_e and spin $\frac{1}{2}$ and mass 1758 m_e are also found. In order to obtain the mass spectrum several approximations are found necessary and it is suggested that provided they are consistent the results might not be very sensitive to the model used for actual calculations.

1. – Introduction.

The wave equation of a point particle quantizes its energy and angular momentum and it is not possible to include any further simultaneous quantization of observables within it. Hence in order to obtain the discrete masses and spins of the elementary particles from a wave equation it is necessary to introduce further independent variables into it. The point model of particles has inherent divergence difficulties and it seems possible to obtain satisfactory results for both problems by postulating that a particle has an internal structure in addition to the external coordinates. Such theories have been developed

(*) Now at the Istituto di Fisica dell'Università di Torino.

by BORN (1) and YUKAWA (2) and they have obtained qualitative spectrum of the masses of the elementary particles. Now as their masses and spins are known better we have attempted to obtain a more quantitative agreement with the known mass spectrum by performing the calculations of YUKAWA (2) in greater detail. The experimental results are not very extensive and are given in the Table I, where the masses of the particles are given in the notation which is generally used to denote them in terms of the electron mass.

TABLE I. – *The measured masses of the unstable elementary particles.*

Spin	Unstable particles m_e
$\frac{1}{2}$ or Fermions	$\mu^{+,-}$ (207), $\varkappa^{+,-}$ (1000), Λ^0 (2185), $\Sigma^{+,-}$ (2390), Ξ^- (2600)
0 or Bosons	$\pi^{+,-}$ (274), π^0 (265), $\tau^{+,-}$ (965), $\vartheta^{+,0,-}$ (965)

To obtain a mass spectrum the simplest procedure is to introduce a suitable potential function in the wave equation for the internal coordinates and one such phenomenological model is given at the end of § 3. The problem of the decay of the unstable elementary particles has been examined in great detail (4) and in order to remain in contact with these developments a field theory model of the structure of the elementary particles is necessary. But as soon as this is done a large number of coupling constants are encountered. Their values are not known and are chosen so that there is agreement with the experimental data. So we have avoided numerical calculations and obtained analytic results. In § 2 we have followed a general lagrangian formulation such that the external coordinate wave equations which are the Dirac equations for particles of given mass, charge and spin remain unaltered. The internal coordinate wave equations are non linear and, after making many approximations, we have solved them in § 3 with the view of obtaining a discrete mass spectrum from

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(1) M. BORN: *Rev. Mod. Phys.*, **21**, 463 (1949).

(2) H. YUKAWA: *Phys. Rev.*, **77**, 219 (1950); **80**, 1047 (1950); **91**, 415, 416 (1953). See also M. FIERZ: *Helv. Phys. Acta*, **23**, 412 (1950); D. R. YENNIE: *Phys. Rev.*, **80**, 1053 (1950); J. RAYSKI: *Nuovo Cimento*, **12**, 815 (1954).

(3) W. B. FOWLER, R. P. SHUT, A. M. THORNDIKE and W. L. WHITTEMORE: *Phys. Rev.*, **98**, 121 (1955); C. DILWORTH, G. P. S. OCCHIALINI and L. SCARSI: *Ann. Rev. of Nuclear Science*, **4**, 271 (1954). See also the references in the footnote (4).

(4) A. PAIS: *Phys. Rev.*, **86**, 663 (1952); *Physica*, **19**, 869 (1953); *Proc. Nat. Acad. Sci.*, **40**, 484, 835 (1954); M. GELL-MANN: *Phys. Rev.*, **92**, 833 (1953); M. GELL-MANN and A. PAIS: *Proceedings of the International Conference of Nuclear Physics, Glasgow* (1954); R. UTIYAMA and W. TOBOCMAN: *Phys. Rev.*, **98**, 780 (1955); R. ROSENDORF, R. STRAHL and G. YEKUTIELI: *Suppl. Nuovo Cimento*, **12**, 247 (1954).

each wave equation and this is given in Table II. These approximations are equivalent to building a simple model of the original equations and provided the model is consistent this procedure has often been useful. The measured and calculated results are compared in § 4.

We should like first of all to give a detailed description of the procedure followed, which will perhaps make the physical picture clearer. The wave function of a particle is considered, in accordance with YUKAWA, to be a function of the external and internal coordinates. The intrinsic properties of a particle, such as its charge, mass and spin, are assumed to be known in the Dirac equation. Now in order to analyse these further we may consider them to be due to the internal motion of the particle. But due to the general validity of the Dirac equation, the internal coordinate wave equation should not affect the results obtained from the Dirac equation, such as for example the scattering cross-sections. It should determine the values of mass, spin and charge only. So we are led to the assumption that the wave function and the wave equation of a particle is the product of a function of external coordinates and a function of internal coordinates. The interactions between particles are chosen with this point in view.

The interaction terms will also be products of internal and external interactions and the coupling constant may also be written as the product of an external and an internal coupling constant. The internal coupling constant will only affect the mass spectrum of the unstable particles. The strength of the interaction between particles which is found in the scattering experiments is completely due to the external factor of the coupling constant and is not considered here. The strength of the internal interaction between the π-mesons and the proton-neutron fields, and between the π-mesons and the electron field is found to depend upon the choice of the model.

Using the above mentioned separability, we have obtained internal coordinate wave equations for every value of spin from a Lagrangian. Since we expect to obtain a mass spectrum from each of them, they are so arranged that the ground states, which have the lowest mass values, represent the stable fundamental particles. Thus for spin 0 the ground state has mass 0, for spin $\frac{1}{2}$ the ground states have masses 0 and $\varkappa_0$, where $\varkappa_0$ is the neutron-proton mass, and the ground state of spin 1 has the mass 0. As the number of spins increases the interaction terms between the particles, and the number of corresponding coupling constants increase. These coupling constants are chosen so that there is agreement with the experimental results. Due to the small number of the latter we shall use the minimum number of coupling constants. The least number of spins that one can consider in a consistent manner is three, that is the spins 0, $\frac{1}{2}$ and 1. This leads to two coupling constants. If necessary further spins can be added when results that cannot be reconciled to the present model are found.

The interaction field in a given field equation may be represented by a simple generating function, and we shall choose such interaction coefficients which lead to the simplest generating functions. We can also linearise the non linear wave equations by the use of these generating functions and the wave equations are put into an easily soluble form. They also show the relation between a field theory model and a purely phenomenological model. This procedure introduces three more arbitrary constants corresponding to the three fields of spin 0, $\frac{1}{2}$ and 1.

Finally the simplest boundary conditions, which are those of a « hard sphere » model where the wave function vanishes at the surface of a sphere of a given radius, are used to obtain the mass eigenvalues. This gives us an additional arbitrary constant. However products of the six arbitrary constants are generally encountered and actually we have four arbitrary constants in this model. Three of these are found from the values of the π, μ and θ-mesons and a reasonable mass spectrum of the elementary particles is obtained. The fourth constant is found to be superfluous at the present stage and is used simply to increase the number of K-mesons.

2. – The Internal Coordinate Wave Equations.

Let $\Psi_F(R_\mu, r_\mu)$ denote the wave function of fermions and $\Psi_B(R_\mu, r_\mu)$ the wave function of bosons in an assembly of particles at the space-time point R_μ. Here r_μ represents the internal coordinate of the particles and describes their structure around R_μ and in the limit $r_\mu \to 0$, we obtain a point model of the particles [2]. These wave functions may be written more explicitly as

$$\Psi_F(R_\mu, r_\mu) = \sum_{\sigma,\varrho} \Psi_{\sigma,\varrho}(R_\mu, r_\mu)\,, \qquad \sigma = \tfrac{1}{2}, \tfrac{3}{2}, \dots \tag{1}$$

$$\Psi_B(R_\mu, r_\mu) = \sum_{\sigma,\varrho} \Psi_{\sigma,\varrho}(R_\mu, r_\mu)\,, \qquad \sigma = 0, 1, \dots \tag{2}$$

where $\Psi_{\sigma,\varrho}(R_\mu, r_\mu)$ is the wave function of the particle whose spin is σ and the magnetic spin quantum states are represented by ϱ.

Then the Lagrangian density of the assembly may generally be written as

$$\begin{aligned} L(R_\mu, r_\mu) = {} & L_F\left(\overline{\Psi}_F(R_\mu, r_\mu), \frac{\partial}{\partial R_\mu}\overline{\Psi}_F(R_\mu, r_\mu), \frac{\partial}{\partial r_\mu}\overline{\Psi}_F(R_\mu, r_\mu), \dots, \Psi_F(R_\mu, r_\mu), \dots\right) + \\ & + L_B\left(\Psi_B(R_\mu, r_\mu), \frac{\partial}{\partial R_\mu}\overline{\Psi}_B(R_\mu, r_\mu), \frac{\partial}{\partial r_\mu}\overline{\Psi}_B(R_\mu, r_\mu), \dots, \Psi_B(R_\mu, r_\mu), \dots\right) + \\ & + I(\overline{\Psi}_F(R_\mu, r_\mu), \Psi_F(R_\mu, r_\mu), \overline{\Psi}_B(R_\mu, r_\mu), \Psi_B(R_\mu, r_\mu), \dots)\,. \end{aligned} \tag{3}$$

According to the present evidence there exist two types of interaction between elementary particles which are called weak and strong interactions according to the strength of the coupling constant. The former, which include the electromagnetic and the β-decay type of interactions and are also used to account for the slow decay of the V-particles, probably contribute only to the fine structure of the mass spectrum of the elementary particles and are neglected here. We can generally take the strong interactions to be of the form

$$I(R_\mu, r_\mu) = \overline{\Psi}_F(R_\mu, r_\mu)(\Lambda_B(R_\mu, r_\mu)\Psi_B(R_\mu, r_\mu))\Psi_F(R_\mu, r_\mu), \tag{4}$$

where

$$\Lambda_B(R_\mu, r_\mu)\Psi_B(R_\mu, r_\mu) = \sum_{\sigma,\varrho} \Lambda_\sigma(R_\mu, r_\mu)\Psi_{\sigma,\varrho}(R_\mu, r_\mu)\,, \qquad \sigma = 0, 1, \ldots \tag{5}$$

and $\Lambda_\sigma(R_\mu, r_\mu)$ is generally a simple differential operator. Then with suitable $L_F(R_\mu, r_\mu)$ and $L_B(R_\mu, r_\mu)$ the Hamiltonian equations become

$$\mathcal{H}_F(R_\mu, r_\mu)\Psi_F(R_\mu, r_\mu) = (\Lambda_B(R_\mu, r_\mu)\Psi_B(R_\mu, r_\mu))\Psi_F(R_\mu, r_\mu) \tag{6}$$

$$\mathcal{H}_B(R_\mu, r_\mu)\Psi_B(R_\mu, r_\mu) = \frac{\delta I}{\delta \Psi_B}\frac{(R_\mu, r_\mu)}{(R_\mu, r_\mu)}\,, \tag{7}$$

where

$$\mathcal{H}_F(R_\mu, r_\mu)\Psi_F(R_\mu, r_\mu) = \sum_{\sigma,\varrho} \mathcal{H}_{\sigma,\varrho}(R_\mu, r_\mu)\Psi_{\sigma,\varrho}(R_\mu, r_\mu)\,, \qquad \sigma = \tfrac{1}{2}, \tfrac{3}{2}, \ldots \tag{8}$$

$$\mathcal{H}_B(R_\mu, r_\mu)\Psi_B(R_\mu, r_\mu) = \sum_{\sigma,\varrho} \mathcal{H}_{\sigma,\varrho}(R_\mu, r_\mu)\Psi_{\sigma,\varrho}(R_\mu, r_\mu)\,, \qquad \sigma = 0, 1, \ldots \tag{9}$$

Here $\mathcal{H}_{\sigma,\varrho}(R_\mu, r_\mu)$ is the Hamiltonian operator of the free particle of spin σ and $\delta I/\delta\Psi_B$ denotes the functional derivative of $I(R_\mu, r_\mu)$ with respect to $\Psi_B(R_\mu, r_\mu)$.

When the mass and spin of a particle are known it is sufficient to consider its external motion and to neglect its internal structure or to consider it to be a point in space time. The mass and spin of a particle may be taken to be its intrinsic properties and be attributed to its internal structure. This suggests that the general equations (6) and (7) are separable in the external and internal coordinates and we may write

$$\begin{cases} \mathcal{H}_{\sigma,\varrho}(R_\mu, r_\mu) = H_{\sigma,\varrho}(R_\mu)h_{\sigma,\varrho}(r_\mu)\,, \\ \Lambda_\sigma(R_\mu, r_\mu) \;\;= \Gamma_\sigma(R_\mu)\gamma_\sigma(r_\mu)\,, \\ \Psi_{\sigma,\varrho}(R_\mu, r_\mu) = \Psi_{\sigma,\varrho}(R_\mu)\psi_{\sigma,\varrho}(r_\mu)\,. \end{cases} \tag{10}$$

Here $\Psi_{\sigma,\varrho}(R_\mu)$ are the generally accepted wave functions of a particle of spin σ

when the wave function of the internal coordinates is taken to be a constant. They form the basis of the vector space of the representation $[l, k]\times[k, l]$ of the full Lorentz group where $\sigma = l + k$ and $\Psi_{\sigma,\varrho}(R_\mu)$ has $2(2l+1)(2k+1)$ components when σ is a half integer and $(2l+1)(2k+1)$ components when σ is an integer. Then since σ is a constant of motion the wave function $\psi_{\sigma,\varrho}(r_\mu)$ may be written as

$$\psi_{\sigma,\varrho}(r_\mu) = \varphi_{\sigma,\varrho}(r) T_\sigma^\varrho(\varphi, \theta_1, \theta_2) \,. \tag{11}$$

Here $T_\sigma^\varrho(\varphi, \theta_1, \theta_2)$ are the orthonormal eigenfunctions of the internal angular momentum operator of the particle. They are the products of the associate spherical harmonic and Tschebyscheff functions. $\varphi_{\sigma,\varrho}(r)$ is a scalar function of r and varies with the spin of the particle. To use the more familiar notation we shall write $\pi(r_\mu)$, $\psi_\varrho(r_\mu)$ and $A_\varrho(r_\mu)$ for $\psi_{0,\varrho}(r_\mu)$, $\psi_{\frac{1}{2},\varrho}(r_\mu)$ and $\psi_{1,\varrho}(r_\mu)$ respectively.

The wave equations (6) and (7) can not be solved without several simplifications and we shall make as many as will bring them into a form that can be solved analytically and gives reasonable results. All these assumptions shall mainly deal with the internal coordinate functions and equations, with which we are concerned here. The wave equations of the external coordinates are well known and can be written in their usual forms and are not discussed here. Actually so little is known about the equations of internal motion that we may consider this work to be the construction of a field model that incorporates the known data and then to find out whether the additional information that is obtained in this process can be satisfactorily explained. The success of this procedure depends upon the insensibility of the results upon the model provided that it is a consistent one.

So first of all we assume that all the known elementary particles have spin 0, $\frac{1}{2}$ or 1. This assumption is not necessary and is made only for simplicity and if necessary other spins can be included in the model. Then using the relations (10) and (11) the equations (6) and (7) become

$$\begin{aligned}
&\sum_\varrho H_{\frac{1}{2},\varrho}(R_\mu) h_{\frac{1}{2},\varrho}(r_\mu) \Psi_{\frac{1}{2},\varrho}(R_\mu)\psi_\varrho(r)\, T_{\frac{1}{2}}^\varrho(\varphi, \theta_1, \theta_2) = \\
&= \sum_{\varrho,\varrho'} [\Gamma_0(R_\mu)\gamma_0(r)\Psi_0(R_\mu)\pi(r)\, T_0^0(\varphi, \theta_1, \theta_2) + \Gamma_1(R_\mu)\gamma_1(r)\Psi_{1,\varrho}(R_\mu)\, A_\varrho(r)\, T_1^\varrho(\varphi, \theta_1, \theta_2)]\cdot \\
&\cdot \Psi_{\frac{1}{2},\varrho'}(R_\mu)\psi_{\varrho'}(r)\, T_{\frac{1}{2}}^{\varrho'}(\varphi, \theta_1, \theta_2) \,,
\end{aligned} \tag{12}$$

$$\begin{aligned}
&H_0(R_\mu) h_0(r_\mu)\Psi_0(R_\mu)\pi(r)\, T_0^0(\varphi, \theta_1, \theta_2) + \\
&\qquad + \sum_\varrho H_{1,\varrho}(R_\mu) h_{1,\varrho}(r_\mu)\Psi_{1,\varrho}(R_\mu)\, A_\varrho(r)\, T_1^\varrho(\varphi, \theta_1, \theta_2) = \\
&= \sum_{\varrho,\varrho'} [\Gamma_0'(R_\mu)\gamma_0'(r) + \Gamma_1'(R_\mu)\gamma_1'(r)]\overline{\Psi}_{\frac{1}{2},\varrho}(R_\mu)\cdot \\
&\qquad \cdot \Psi_{\frac{1}{2},\varrho'}(R_\mu)\psi_\varrho(r)\psi_{\varrho'}(r)\, T_{\frac{1}{2}}^\varrho(\varphi, \theta_1, \theta_2)\, T_{\frac{1}{2}}^{\varrho'}(\varphi, \theta_1, \theta_2).
\end{aligned} \tag{13}$$

Here $\Gamma_0'(R_\mu)$, $\gamma_0'(r)$, $\Gamma_1'(R_\mu)$ and $\gamma_1'(r)$ represent the operators that are obtained in place of $\Gamma_0(R_\mu)$ $\gamma_0(r)$, $\Gamma_1(R_\mu)$ and $\gamma_1(r)$ respectively when the functional derivative in (7) is performed. We can write the products of $T_\sigma^\varrho(\varphi, \theta_1, \theta_2)$ on the right hand side of (12) and (13) as a linear series of $T_\sigma^\varrho(\varphi, \theta_1, \theta_2)$ whose coefficients can be expressed in terms of the Clebsch-Gordan coefficients, and we obtain

$$T_{\frac{1}{2}}^{\varrho}(\varphi, \theta_1, \theta_2) T_{\frac{1}{2}}^{\varrho'}(\varphi, \theta_1, \theta_2) = \sum_{\sigma=1,0;\varrho''} c(\sigma, \varrho'', \varrho, \varrho') T_\sigma^{\varrho''}(\varphi, \theta_1, \theta_2) , \tag{14}$$

and

$$T_1^\varrho(\varphi, \theta_1, \theta_2) T_{\frac{1}{2}}^{\varrho'}(\varphi, \theta_1, \theta_2) = \sum_{\sigma=\frac{1}{2},\frac{3}{2};\varrho''} c(\sigma, \varrho'', \varrho, \varrho') T_\sigma^{\varrho''}(\varphi, \theta_1, \theta_2) . \tag{15}$$

However the left hand side of (12) does not contain a term of the type $T_{\frac{3}{2}}^\varrho(\varphi, \theta_1, \theta_2)$ and in order to maintain the self consistency of the model we will choose such a $\gamma_1(r) A_\varrho(r)$ that it is a small and negligible quantity.

Then utilizing the orthonormal properties of $T_\sigma^\varrho(\varphi, \theta_1, \theta_2)$ and the external wave functions, we obtain

$$\begin{aligned} H_{\frac{1}{2}}(R_\mu) h_{\frac{1}{2},\varrho}(r_\mu) \Psi_{\frac{1}{2},\varrho}(R_\mu) \psi(r) T_{\frac{1}{2}}^\varrho(\varphi, \theta_1, \theta_2) &= \\ = \Gamma_0(R) \gamma_0(r) \Psi_0(R_\mu) \pi(r) \Psi_{\frac{1}{2},\varrho}(R_\mu) \psi(r) T_{\frac{1}{2}}^\varrho(\varphi, \theta_1, \theta_2) , \end{aligned} \tag{16}$$

$$\begin{aligned} H_0(R_\mu) h_0(r_\mu) \Psi_0(R_\mu) \pi(r) T_0^0(\varphi, \theta_1, \theta_2) &= \\ = [\Gamma_0'(R)\gamma_0'(r) + \Gamma_1'(R)\gamma_1'(r)] (\bar\Psi_{\frac{1}{2},\varrho}(R_\mu) \Psi_{\frac{1}{2},\varrho'}(R_\mu))_0 \psi(r)^2 c(0, 0) T_0^0(\varphi, \theta_1, \theta_2) , \end{aligned} \tag{17}$$

$$\begin{aligned} H_1(R_\mu) h_1(r_\mu) \Psi_{1,\varrho}(R_\mu) A_\varrho(r) T_1^\varrho(\varphi, \theta_1, \theta_2) &= \\ = [\Gamma_0'(R)\gamma_0'(r) + \Gamma_1'(R)\gamma_1'(r)] (\bar\Psi_{\frac{1}{2},\varrho}(R_\mu) \Psi_{\frac{1}{2},\varrho'}(R_\mu))_1 \psi(r)^2 c(1, \varrho) T_1^\varrho(\varphi, \theta_1, \theta_2) , \end{aligned} \tag{18}$$

where $(\bar\Psi_{\frac{1}{2},\varrho}(R_\mu)\Psi_{\frac{1}{2},\varrho'}(R_\mu))_\sigma$ is the σ spin component of the r.h.s. external functions and $c(\sigma, \varrho)$ are the constant coefficients obtained after summation. We expect these wave equations to be separable and can readily write (16) in a separable form in the external and internal coordinates. It will be seen in the next section that further assumptions shall make (17) and (18) separable also. We will give their separated forms here by writing

$$\Gamma_0'(R)\gamma_0'(r) + \Gamma_1'(R)\gamma_1'(r) = \Gamma'(R)\gamma'(r) \tag{19}$$

and the equations (16), (17) and (18) may be written as

$$\text{(20)}\qquad \begin{cases} H_{\frac{1}{2}}(R_\mu)\Psi_{\frac{1}{2},\varrho}(R_\mu) = \Gamma_0(R)\Psi_0(R_\mu)\Psi_{\frac{1}{2},\varrho}(R_\mu)\,, \\ H_0(R_\mu)\Psi_0(R_\mu) \quad = \Gamma'(R)(\overline{\Psi}_{\frac{1}{2},\varrho}(R_\mu)\Psi_{\frac{1}{2},\varrho'}(R_\mu))_0\,, \\ H_1(R_\mu)\Psi_{1,\varrho''}(R_\mu) = \Gamma'(R)(\overline{\Psi}_{\frac{1}{2},\varrho}(R_\mu)\Psi_{\frac{1}{2},\varrho'}(R_\mu))_{1,\varrho''}\,, \end{cases}$$

$$\text{(21)}\qquad h_{\frac{1}{2}}(r_\mu)\psi(r)T_{\frac{1}{2}}^{\varrho}(\varphi,\theta_1,\theta_2) \doteq \gamma_0(r)\pi(r)\psi(r)T_{\frac{1}{2}}^{\varrho}(\varphi,\theta_1,\theta_2)\,,$$

$$\text{(22)}\qquad h_0(r_\mu)\pi(r)T_0^0(\varphi,\theta_1,\theta_2) = c(0,0)\gamma'(r)\psi(r)^2T_0^0(\varphi,\theta_1,\theta_2)\,,$$

$$\text{(23)}\qquad h_1(r_\mu)A_\varrho(r)T_1^{\varrho}(\varphi,\theta_1,\theta_2) = c(1,\varrho)\gamma'(r)\psi(r)^2T_1^{\varrho}(\varphi,\theta_1,\theta_2)\,.$$

From (20) it is clear that we have attempted to leave unaltered the wave equations for the external coordinates. We note that they are local wave equations and contain the divergent renormalization terms. A procedure, based on non-local interaction fields which are functions of the sum of external and internal coordinates, which eliminates the divergences of the field equations and leaves only finite renormalization terms, has been given before and can be included here. But in this paper we are interested only in obtaining a mass spectrum and consider the masses in the internal coordinate wave equations to be the dressed masses. The equations (21), (22) and (23) describe the internal motion of the particles of spin $\frac{1}{2}$, 0 and 1 respectively, and from each of them we shall obtain a discrete mass spectrum for particles of that spin and the ground state shall represent the stable state of minimum mass. Thus for spins 0 and 1 the ground states will have masses zero. We shall give the interaction term an explicit mass factor in order to be able to accomplish this. But for spin $\frac{1}{2}$, depending upon the charge, the ground state can have mass values of zero or $\varkappa_0$ and again we shall modify $h_{\frac{1}{2}}(r_\mu)$ and the interaction term in a suitable manner.

3. – The Eigenvalues of the Wave Equations.

We shall now proceed to solve the equations (21), (22) and (23) by making further approximations and shall obtain analytic expressions for their eigenfunctions and eigenvalues. So far the field quantities $\psi_{\sigma,\varrho}(r_\mu)$ have not been specified. Now since a discrete mass spectrum for the particles is expected from each wave equation, let the corresponding eigenfunctions be represented by $\psi_{\sigma,\varrho,\varkappa_m}(r_\mu)$. Then the interaction field in the equations (21), (22) and (23) may in general be represented by

$$\text{(24)}\qquad \psi_{\sigma,\varrho}(r_\mu) = \sum_m \alpha_\sigma(\varkappa_m)\psi_{\sigma,\varrho,\varkappa_m}(r_\mu)\,.$$

Using the above specification of the field we shall also find it possible to linearize the wave equations and to obtain simple solutions for them. Here $\alpha_\sigma(\varkappa_m)$ is the interaction coefficient of the interaction of a given field with the field $\psi_{\sigma,\varrho,\varkappa_m}(r_\mu)$ and in building a model we may choose suitable values for them. Furthermore let

$$(25)\qquad \begin{cases} h_0(r_\mu) = h_1(r_\mu) = \Box^2(r_\mu) - \varkappa^2, \\ h_{\frac{1}{2}}(r_\mu) = \Box^2(r_\mu) - \varkappa(\varkappa - \varkappa_0\,\delta_{z(z-1),0}), \\ \gamma_0(r) \;= \varkappa\lambda_0 \dfrac{z(z-1)}{2}, \quad \gamma_1(r) = \varkappa\lambda_1 r\dfrac{\partial}{\partial r}, \end{cases}$$

where $\varkappa$ is a mass operator defined by the relation

$$(26)\qquad \varkappa\psi_{\sigma,\varrho}(r_\mu) = \sum_m \varkappa_m \psi_{\sigma,\varrho,\varkappa_m}(r_\mu),$$

and λ_0 and λ_1 are arbitrary coupling constants. $\varkappa_0$ is the mass of the proton and the term $\varkappa\varkappa_0\,\delta_{z(z-1),0}$ in $h_{\frac{1}{2}}(r_\mu)$ and the factor $z(z-1)/2$ in $\gamma_0(r)$ have been added to account for the existence of the four stable particles, neutrino, electron, neutron, and proton for spin $\frac{1}{2}$. Of course the mass of the electron is taken to be zero and the masses of neutron and proton are taken to be equal to each other.

Then using the relation (25), the equation (21), (22) and (23) become

$$(27)\qquad (\Box^2(r_\mu) - \varkappa^2)\psi^- T_{\frac{1}{2}}^{\varrho}(\varphi, \theta_1, \theta_2) = \varkappa\left(\lambda_0\pi(r) + \lambda_1(r)\frac{\partial}{\partial r}A_\varrho(r)\right)\psi^-(r)T_{\frac{1}{2}}^{\varrho}(\varphi, \theta_1, \theta_2),$$

$$(28)\qquad (\Box^2(r_\mu) - \varkappa(\varkappa - \varkappa_0))\psi^{+,0}T_{\frac{1}{2}}^{\varrho}(\varphi, \theta_1, \theta_2) = \varkappa\left(\lambda_1 r\frac{\partial}{\partial r}A_\varrho(r)\right)\psi^{+,0}(r)T_{\frac{1}{2}}^{\varrho}(\varphi, \theta_1, \theta_2),$$

$$(29)\qquad (\Box^2(r_\mu) - \varkappa^2)\,\pi(r)T_0^{0}(\varphi, \theta_1, \theta_2) = c(0, 0)\frac{\varkappa\gamma'(r)\,\psi(r)^2}{\pi(r)}\pi(r)T_0^{0}(\varphi, \theta_1, \theta_2),$$

$$(30)\qquad (\Box^2(r_\mu) - \varkappa^2)A_\varrho(r)T_1^{\varrho}(\varphi, \theta_1, \theta_2) = c(1, \varrho)\frac{\varkappa\gamma'(r)\,\psi(r)^2}{A_\varrho(r)}A_\varrho(r)T_1^{\varrho}(\varphi, \theta_1, \theta_2),$$

where we have taken

$$(31)\qquad \psi(r) = \psi^-(r)\,\delta_{z+1,0} + \psi^{+,0}(r)\,\delta_{z(z-1),0}\,.$$

(27) represents the wave equation for the electron and is obtained by putting $z = -1$ in (21). Similarly (28) is obtained by putting $z = 0$ or 1 in (21) and represents the wave equation for neutrino on one hand and neutron

proton on the other. To solve these equations further we shall make the « hard sphere » approximation that is we shall assume that the wave functions are zero for $r \geqslant a$. We note that this assumption affects the unperturbed Hamiltonians also and we obtain a discrete spectrum for the unperturbed Hamiltonians as well.

In order to find the eigenfunctions of equations (27) to (30) we must know $\pi(r)$, $\psi(r)$ and $A_\varrho(r)$ which occur in the interaction terms and which are functions of the eigenfunctions themselves. Now the sums of the eigenfunctions can generally be expressed by a simple generating function and we will utilize this idea in the relation (24) and choose such $\alpha_\sigma(\varkappa_m)$'s that specially simple values of $\pi(r)$, $\psi(r)$ and $A(r)$ are obtained. So let

$$\left\{\begin{aligned} &\pi(r) \equiv \sum_m \alpha_0(\varkappa_m)\pi_{\varkappa_m}(r) = c_0 && \text{for } r < a\,, \\ &\psi(r) = c_{\frac{1}{2}},\ A_\varrho(r) = c_1(\varrho) && \\ &\text{and} && \\ &\pi(r) = \psi(r) = A_\varrho(r) = 0 && \text{for } r > a\,, \end{aligned}\right. \tag{32}$$

where c_0, $c_{\frac{1}{2}}$ and c_1 are constant numbers. Then the equations (27) to (30) become

$$\left\{\begin{aligned} &(\Box^2(r_\mu) - \varkappa^2)\,\psi_\varkappa^-(r)\,T_{\frac{1}{2}}^\varrho(\varphi, \theta_1, \theta_2) &&= \varkappa\lambda'\psi_\varkappa^-(r)\,T_{\frac{1}{2}}^\varrho(\varphi, \theta_1, \theta_2)\,, \\ &(\Box^2(r_\mu) - \varkappa(\varkappa - \varkappa_0))\,\psi_\varkappa^{+,0}(r)T_{\frac{1}{2}}^\varrho(\varphi, \theta_1, \theta_2) &&= 0, \\ &(\Box^2(r_\mu) - \varkappa^2)\pi_\varkappa(r)\,T_0^0(\varphi, \theta_1, \theta_2) &&= \varkappa\lambda''\pi_\varkappa(r)\,T_0^0(\varphi, \theta_1, \theta_2)\,, \\ &(\Box^2(r_\mu) - \varkappa^2)\,A_{\varrho,\varkappa}(r)\,T_1^\varrho(\varphi, \theta_1, \theta_2) &&= \varkappa\lambda'''(\varrho)\,A_{\varrho,\varkappa}(r)\,T_1^\varrho(\varphi, \theta_1, \theta_2)\,, \end{aligned}\right. \tag{33}$$

where λ', λ'' and λ''' are arbitrary constants. Those solutions of these equations which are finite for $r = 0$ are

$$\left\{\begin{aligned} &\pi_{\varkappa_m}(r) &&= N_0(\varkappa_m)rJ_1(\sqrt{\varkappa_m^2 - \lambda''\varkappa_m}\,r) \\ &\psi_{\varkappa_m}^-(r) &&= N_{\frac{1}{2}}^-(\varkappa_m)rJ_{\frac{3}{2}}(\sqrt{\varkappa_m^2 - \lambda'\varkappa_m}\,r) \\ &\psi_{\varkappa_m}^{+,0}(r) &&= N_{\frac{1}{2}}^+(\varkappa_m)rJ_{\frac{3}{2}}(\sqrt{\varkappa_m^2 - \varkappa_0\varkappa_m}\,r) \\ &A_{\varrho,\varkappa_m}(r) &&= N_1(\varkappa_m)rJ_2(\sqrt{\varkappa_m^2 - \lambda'''(\varrho)\varkappa_m}\,r) \end{aligned}\right. \tag{34}$$

where the $N(\varkappa_m)$'s are the normalizing factors. Since these wave functions become zero when $r = a$, we note that, for instance, $\sqrt{\varkappa_m - \lambda''\varkappa_m}\,a$ is the m-th zero of $J_1(\sqrt{\varkappa^2 - \lambda''\varkappa}\,a)$ and the boundary condition gives us the desired discrete mass spectrum.

A mass spectrum has been calculated and is given in Table II. Here the constants λ', λ'' and a have the values 207 m_e, 270 m_e and .004679 m_e^{-1} respectively and are chosen so that the μ-meson has spin $\frac{1}{2}$ and mass 207 m_e, the π-meson has spin 0 and mass 270 m_e and the θ-meson has spin 0

TABLE II. – *Calculated masses of the unstable particles.*

Spin	Charge	Masses of the unstable particles				m_e
$\frac{1}{2}$	+, –	207	1070	1758	2600	.
	+, 0, –	2247	2807	.	.	.
0	+, 0, –	270	965	1640	2287	.
1	+, 0, –	1000	.	.	.	.

and mass 965 m_e. We can expect to find the charge conjugates of particles of spin $\frac{1}{2}$ also. We have neglected negative masses as the mass measurements are made with respect to the stable masses and they are considered to have the lowest mass state here. The value of $\lambda'''(\varrho)$ may be so chosen that the first excited mass state of spin 1 has the mass 1000 m_e and may be taken to be a K-meson. However specific data about such particles is lacking and we can modify its value to suit the circumstances. In the next section these results are compared with the measured results of Table I.

Now we should like to give a purely phenomenological model with which this investigation started. Let the excited states of the stable particles electrons and protons be fermions and let them originate from a potential of the form $1/r$. Let the mass term of the internal coordinate Hamiltonian $h_F(r_\mu)$ be a quadratic in $\varkappa$ suggesting an elastic structure. Then the fermion masses may be considered to be quantized in such a way that

$$\text{(35)} \qquad \text{Fermions:} \qquad \varkappa(\varkappa - \varkappa_0\delta_{z(z-1),0}) = A(n - \tfrac{1}{2})^2$$

where the total quantum number n is the sum of the radial quantum number n_r and the spin quantum number σ, and $n_r = 0, 1, \ldots$. Similarly we may write for the bosons

$$\text{(36)} \qquad \text{Bosons:} \qquad \varkappa^2 = B(n-1)^2\,.$$

Let these assemblies be coupled. Then in analogy with the harmonic oscil-

lators ([5]), the spectrum of the fermion and boson masses becomes

$$\text{(37)}\quad \text{Fermions:}\ \varkappa(\varkappa-\varkappa_0\delta_{z(z-1),0}) = A(n-\tfrac{1}{2})^2 - C(n-\tfrac{1}{2})(n-1)\,, \quad n=\tfrac{1}{2},\tfrac{3}{2},\ldots.$$

$$\text{(38)}\quad \text{Bosons:}\ \varkappa^2 = B(n-1)^2 + C(n-\tfrac{1}{2})(n-1)\,, \quad n=0,1,\ldots.$$

Fitting A, B and C to the data $\varkappa = 207\ \mathrm{m_e}$ for $n = 1.5$, $\varkappa = 965\ \mathrm{m_e}$ for $n=0$ and $\varkappa=270\ \mathrm{m_e}$ for $n=2$, we obtain $A=-.3862\cdot 10^6\ \mathrm{m_e^2}$, $B=1.0362\cdot 10^6\ \mathrm{m_e^2}$ and $c = -.8582\cdot 10^6\ \mathrm{m_e^2}$ and the spectrum is given in the Table III.

TABLE III. – *Masses of the elementary particles from a purely phenomenological model.*

	Charge \ n	$\frac{1}{2}$	$1\frac{1}{2}$	$2\frac{1}{2}$	$3\frac{1}{2}$	$4\frac{1}{2}$	. $\mathrm{m_e}$
	+, −	0	207	1001	1721	2415	.
Fermions	+, 0, −	1837	1860	2283	2869	.	.
	Charge \ n	0	1	2	3	4	5
Bosons	+, 0, −	965	0	270	1074	1797	2516

4. – Conclusion.

We have built a field theory model of the equation of motion of the internal coordinates such that corresponding to each spin there is a wave equation. The mass state corresponding to the mass of the stable particle is considered to be the ground state and states that correspond to negative masses are neglected on physical grounds. We then interpret the excited mass states to be the unstable elementary particles. There are four arbitrary constants and their values are chosen so that there exists a meson of spin $\frac{1}{2}$ and mass 207 $\mathrm{m_e}$, a meson of spin 0 and mass 270 $\mathrm{m_e}$, a meson of spin 0 and mass 965 $\mathrm{m_e}$ and a meson of spin 1 and mass 1000 $\mathrm{m_e}$, and we can check our theory by comparing the rest of the Table II with the experimental results of Table I.

While there appears to be a general agreement, for instance the gap in masses between 270 and 950 $\mathrm{m_e}$ is reproduced and the mass of Λ^0 is obtained with sufficient accuracy, there appear to be several discrepancies, such as the existence of masses 1640 and 1758 and the triplet isotopic state for Λ-particles rather than the generally preferred singlet state. However the data on these masses is insufficient at present and the cross-section for the production of π (965) and ψ (1070) will certainly be larger than the cross-section for the

([1]) C. KITTEL: *Introduction to Solid State Physics* (New York, 1953).

production of $\pi(1640)$ and $\psi(1758)$. It is interesting to note that the value which we obtain for the mass $\psi^{+,0,-}(2247)$ is intermediate between the mass values of Λ^0 and $\Sigma^{+,-}$. We have also neglected all the fine structure effects such as the difference between the masses of π^0 and $\pi^{+,-}$. Due to insufficient data we have not been able to assign a proper role to the unstable particles of spin 1 in this classification and we can temporarily use them to increase the number of K-mesons.

This model has been so built that the internal strong interactions vanish for the ground states and it seems to be in accordance with the general interpretation for the ground states of zero masses. But if we wish to have a larger internal interaction for the neutron and proton we note that it will contribute also to their masses and the equation in (33) for the internal motion of neutrons and protons should then be replaced by

$$(39)\qquad (\Box^2(r_\mu)-\varkappa(\varkappa-(\varkappa_0-\xi)))\psi^{+,0}(r)\,T_{\frac{1}{2}}^{\varrho}(\varphi,\theta_1,\theta_2)=\varkappa\lambda'\psi^{+,0}\,T_{\frac{1}{2}}^{\varrho}(\varphi,\theta_1,\theta_2),$$

where ξ is a constant and is to be so chosen that the neutron-proton ground state has the mass $\varkappa_0$. We obtain the value of 207 m_e for it and the other results remain unaltered. These considerations also show again that we have only attempted to construct a consistent model for the mass spectrum of the unstable particles.

We can also, if it is necessary, modify our model further, and the isotopic spin of the particle $\psi^{+,0,-}(2247)$ can be altered so that it becomes $\psi^0(2247)$. To do this we can write new Lagrangian densits such that the wave function $\psi(r)$ becomes

$$(40)\qquad \psi(r)=rJ_{\frac{3}{2}}(\sqrt{f_F(z)[\varkappa(\varkappa-\varkappa_0\,\delta_{z(z-1),0}-\varkappa\lambda z(z-1)/2)]}\,r)\,.$$

Now let

$$(41)\qquad f_F(z)=\begin{matrix}1 & \text{for} & z=-1 \text{ or } 0\\ \infty & \text{for} & z=1\end{matrix}$$

and the above mentioned modification is obtained. If necessary other isotopic spin adjustments can easily be made.

* * *

The author is grateful to Professor G. WATAGHIN for kind hospitality at the Istituto di Fisica dell'Università and for helpful discussions.

FONDAZIONE FRANCESCO SOMAINI
PRESSO IL TEMPIO VOLTIANO, A COMO

BANDI DI CONCORSI
AL PREMIO E ALLA BORSA PER IL 1958

Con lo scopo di premiare e incoraggiare nel nome di ALESSANDRO VOLTA gli studi di Fisica in Italia, la « Fondazione Francesco Somaini », presso il Tempio Voltiano a Como, indice i seguenti concorsi.

A) **Concorso al "Premio Triennale per la Fisica Francesco Somaini" per il 1958.** di L. 1 500 000 (un milione e cinquecentomila) nette, da assegnarsi al concorrente che, fra quelli che la Commissione Giudicatrice giudicherà in senso assoluto meritevoli del premio per i risultati conseguiti nello studio della Fisica durante il Triennio 1° Luglio 1955-30 Giugno 1958, sia, a parere della Commissione stessa, il più meritevole.

B) **Concorso alla "Borsa Francesco Somaini per lo studio della Fisica" per il 1958.** di L. 750 000 (settecentocinquantamila) nette, da assegnarsi al concorrente che, tra quelli che la Commissione Giudicatrice giudicherà in senso assoluto meritevoli della Borsa, verrà dalla Commissione stessa giudicato il più meritevole, sia per titoli, preparazione scientifica, lavori già svolti e risultati già conseguiti nella Fisica, sia anche per il vantaggio che gli studi, per i quali è richiesta la Borsa, possono portare allo sviluppo della Fisica, in Italia.

1. – Ad entrambi i Concorsi possono prendere parte singolarmente i cittadini, d'ambo i sessi, italiani e svizzeri del Canton Ticino purchè di stirpe italiana. Sono esclusi dal Concorso i membri della Commissione Amministratrice e della Commissione Scientifica della « Fondazione Francesco Somaini ».

2. – Le norme particolareggiate dei singoli Concorsi verranno pubblicate in apposito volantino che potrà essere richiesto dagli interessati alla Segreteria della Fondazione presso il Tempio Voltiano a Como.

3. – La domanda, i documenti, i lavori, ecc., presentati dai singoli concorrenti dovranno pervenire, tra il 1° Gennaio e le ore 12 del 1° Luglio 1958, alla Commissione Amministratrice della « Fondazione Francesco Somaini » a Como presso il Tempio Voltiano.

4. – Il Premio Triennale per la Fisica potrà essere anche conferito a uno studioso che non abbia preso parte al Concorso, ma sia stato segnalato da un Membro della Commissione Giudicatrice, con proposta motivata, come meritevole di particolare considerazione oppure ritenuto degno di premio dalla Commissione Giudicatrice, indipendentemente da ogni segnalazione.

* * *

La procedura dei suddetti Concorsi è regolata secondo lo Statuto della Fondazione, il quale è ostensibile a Como presso il Tempio Voltiano ed è depositato negli atti del Notaio Dr. Raoul Luzzani di Como.

Como, dal Tempio Voltiano, il giorno 6 Agosto 1955.

Il Segretario Conservatore del Tempio
CESARE MORLACCHI

Il Presidente Sindaco di Como
PAOLO PIADENI

[Il bando in esteso di detti Concorsi è stato pubblicato nel fascicolo di Gennaio del *Nuovo Cimento* 1956].

SOCIETÀ ITALIANA DI FISICA

CONCORSO SPECIALE A UN PREMIO PER STUDI SUI MATERIALI SEMICONDUTTORI

Prendendo occasione dalla prossima inaugurazione a Forlì del Centro di Ospitalità per scrittori, artisti e scienziati, fondato, in memoria di Livio e Maria Garzanti, dal figlio, l'Editore Aldo Garzanti, questi ha istituito un premio scientifico, del quale ha fissato la natura e lo scopo con la sequente dichiarazione:

« Considerando la grande importanza che in questi ultimi anni hanno acquistato, sia nel campo strettamente scientifico sia nelle applicazioni, i materiali semiconduttori, viene istituito un premio di un milione di lire a favore di uno studioso italiano che, entro il 31 Maggio 1956, abbia presentato una memoria, nella quale, premesso un breve riassunto sulla Fisica di questi materiali, siano indicate, nei termini più concreti, le possibilità di fabbricazione e di impiego di essi in Italia, con riferimento alle materie prime qui esistenti, ai costi e alle condizioni della nostra industria. Sarà tenuto conto dei contributi scientifici e tecnici personali che il candidato abbia eventualmente portato alla conoscenza delle proprietà e delle applicazioni di questi materiali. Il premio sarà assegnato alla migliore tra le memorie giudicate degne dalla apposita Commissione. La proprietà letteraria e il diritto di eventuale pubblicazione della memoria vincitrice spettano all'Editore Garzanti ».

La Società Italiana di Fisica, richiesta dall'Editore Garzanti, ha ben volentieri acconsentito, applaudendo alla iniziativa di lui, a patrocinare il Concorso e curare tutte le operazioni che esso comporta: e quindi d'intesa con l'Editore stesso ha formulato il seguente Bando.

1. – È aperto un concorso a un premio di L. 1000000 (un milione) intitolato « Premio Livio e Maria Garzanti », per studi sui materiali semiconduttori.

2. – Possono partecipare al Concorso le persone di nazionalità italiana che non facciano parte della Commissione esaminatrice.

3. – Le domande di ammissione al Concorso, redatte su carta libera e recanti le generalità e il recapito del concorrente, dovranno pervenire alla Presidenza della Società Italiana di Fisica (Milano, Via Saldini, 50) entro il 31 Maggio 1956.

Alla domanda il concorrente dovrà unire: *a*) la dichiarazione firmata che egli è di nazionalità italiana; *b*) cinque copie (dattilografate) di una memoria originale, da lui compilata, nella quale, premesso un breve riassunto sulla Fisica dei materiali semiconduttori, siano indicate, nei termini più concreti, le possibilità di fabbricazione e d'impiego di essi in Italia, con riferimento alle materie prime qui esistenti, ai costi e alle condizioni della nostra industria; *c*) quei titoli, documenti e pubblicazioni (queste in cinque copie) che possano attestare il contributo portato dal concorrente alla conoscenza delle proprietà fisiche e alle applicazioni tecniche e pratiche dei materiali semiconduttori; *d*) sei copie (dattiloscritte) dell'elenco completo di tutte le carte presentate.

4. – La Commissione giudicatrice del concorso è costituita da cinque membri, di cui quattro nominati dal Consiglio di Presidenza della Società Italiana di Fisica e uno dall'Editore Aldo Garzanti in sua rappresentanza.

5. – Il Premio sarà assegnato alla migliore tra le memorie giudicate degne dalla Commissione. La proprietà letteraria e il diritto dell'eventuale pubblicazione della memoria vincitrice spettano all'Editore Aldo Garzanti.

6. – Il giudizio della Commissione è inappellabile.

7. – La proclamazione del vincitore avrà luogo, per quanto in tempo, alla inaugurazione del Centro di Ospitalità di Forlì, altrimenti, insieme con il conferimento del Premio stesso al Congresso Internazionale di Fisica che si terrà nel prossimo Settembre a Torino per celebrare il grande fisico e chimico italiano Amedeo Avogadro nel centenario della sua morte.

Il Segretario: G. C. Dalla Noce — *Il Presidente*: G. Polvani

RIASSUNTO (*)

Partendo dalla teoria non locale di Yukawa per lo spettro di massa delle particelle elementari, si costruisce un modello di teoria di campo. Si trova in corrispondenza di ogni spin un'equazione di campo il cui stato fondamentale rappresenta la massa della particella stabile dotata di quello spin e i suoi stati eccitati si interpretano come la particelle elementari instabili. Ammettendo di scegliere valori adatti per le costanti di accoppiamento arbitrarie, la teoria appare in soddisfacente accordo con i caratteri generali dello spettro di massa e fornisce una massa di 2247 m_e per le particelle $\Lambda^{+,0,-}$ con spin $\frac{1}{2}$. Si trovano anche particelle con spin 0 e masse 1640 m_e, 2287 m_e e spin $\frac{1}{2}$ e massa 1758 m_e. Per ottenere lo spettro di massa sono necessarie alcune approssimazioni e si presume che, purchè queste siano compatibili, i risultati non siano molto sensibili al modello usato per l'effettiva esecuzione dei calcoli.

(*) *Traduzione a cura della Redazione.*

La condizione supplementare del campo di Stückelberg.

P. Bocchieri e A. Loinger

Istituto di Fisica dell'Università - Pavia
Istituto Nazionale di Fisica Nucleare - Sezione di Milano

(ricevuto il 19 Gennaio 1956)

Riassunto. — L'usuale condizione supplementare della teoria del campo di Stückelberg conduce a difficoltà simili a quelle presentate in elettrodinamica dalla condizione di Fermi. Si mostra che, anche in questo caso, l'introduzione di una metrica indefinita permette di evitare completamente tali difficoltà. Particolarmente degno di nota è il comportamento del campo di Stückelberg complesso, per il quale la condizione supplementare rimane, a metrica indefinita introdotta, formalmente immutata, pur essendo naturalmente diverso il significato degli operatori che in essa figurano.

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1. – Le difficoltà cui danno luogo le condizioni supplementari della teoria *quantistica* dei campi sono state finora indagate e soddisfacentemente risolte nel solo caso dell'elettrodinamica (1) mediante l'introduzione, nello spazio dei vettori astratti, di una metrica indefinita nel senso di Dirac e Pauli (2,3). Viene spontaneo chiedersi se un opportuno uso della metrica indefinita possa consentire il superamento delle difficoltà connesse colle condizioni supplementari di altre teorie. Per rispondere a questa domanda, abbiamo pensato di

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(1) K. Bleuler: *Helv. Phys. Acta*, **23**, 567 (1950).

(2) W. Pauli: *Rev. Mod. Phys.*, **15**, 175 (1943). La metrica indefinita è stata recentemente usata da G. Källén e W. Pauli (*Dan. Mat. Fys. Medd.*, **30**, No. 7 (1955)) per dare una formulazione matematica rigorosa della teoria del modello di Lee.

(3) G. Källén (*Die « große Eichinvarianz »*, non pubblicato) è recentemente riuscito a evitare il problema della condizione supplementare dell'elettrodinamica, scrivendo delle equazioni nelle quali il « gauge » dei potenziali elettromagnetici non è precisato. Le equazioni di Källén sono, per la teoria di matrice S, completamente equivalenti alla usuale formulazione con metrica indefinita.

prendere in esame la più generale formulazione della teoria del campo bosonico vettoriale, data da STÜCKELBERG [4], nella quale interviene appunto una condizione supplementare. Abbiamo trattato i due casi di campo reale e di campo complesso, dedicando una maggiore attenzione al primo, la teoria del quale, in quanto (perturbativamente) rinormalizzabile, ha forse un interesse fisico un po' maggiore.

2. – Consideriamo dapprima il campo di Stückelberg *reale*; nel caso di assenza di interazioni le equazioni di moto e i commutatori sono:

$$(\Box - \mu^2)A_\nu = 0 \; ; \qquad (\Box - \mu^2)B = 0 \; ;$$

$$(a) \qquad [A_\lambda(x), A_\nu(x')] = i\,\delta_{\lambda\nu}\,\Delta(x - x') \; ;$$

$$(b) \qquad [B(x), B(x')] = i\,\Delta(x - x') \; ;$$

$$(c) \qquad [A_\lambda(x), B(x')] = 0 \, .$$

La condizione supplementare usualmente postulata è:

$$(1) \qquad \left(\frac{\partial A_\nu}{\partial x_\nu} + \mu B\right)|\rangle = 0 ;$$

da cui:

$$(1') \qquad \langle|\left(\frac{\partial A_\nu}{\partial x_\nu} + \mu B\right) = 0 \, .$$

Analogamente a quanto si verifica in elettrodinamica per la condizione di Fermi [5], anche la (1) è però banalmente incompatibile coi commutatori. Infatti, derivando la (a) rispetto a x_λ e prendendo i valori di aspettazione di entrambi i membri, si ha:

$$\langle|\left[\frac{\partial A_\lambda(x)}{\partial x_\lambda}, A_\nu(x')\right]|\rangle = i\,\frac{\partial}{\partial x_\nu}\,\Delta(x - x') ;$$

ma, per le (1), (1′), il primo membro è nullo. Considerazioni analoghe valgono per la (b) e per la (c).

Se, in luogo della (1), si imponesse la:

$$(2) \qquad \left(\frac{\partial A_\nu^{(+)}}{\partial x_\nu} + \mu B^{(+)}\right)|\rangle = 0 \, ,$$

(4) W. PAULI: *Rev. Mod. Phys.*, **13**, 217 (1941); R. J. GLAUBER: *Prog. Theor. Phys.*, **9**, 295 (1953); **10**, 690 (1953).

(5) P. BOCCHIERI e A. LOINGER: *Nuovo Cimento*, **3**, 221 (1956).

(ove con $^{(+)}$ si intende, al solito, la parte a frequenze positive) difficoltà di questo genere non si presenterebbero; senonchè la (2), al pari della (1), rende di norma infinita il vettore di stato del sistema [6]. Di ciò ci si convince ragionando come segue.

Consideriamo gli sviluppi di Fourier di $A_\nu(x)$ e $B(x)$:

$$A_\nu(x) = \frac{1}{\sqrt{V}} \sum_{\boldsymbol{k}} \frac{1}{\sqrt{2\omega(\boldsymbol{k})}} \{a_\nu(\boldsymbol{k}) \exp[i(kx)] + a_\nu^*(\boldsymbol{k}) \exp[-i(kx)]\} ;$$

$$B(x) = \frac{1}{\sqrt{V}} \sum_{\boldsymbol{k}} \frac{1}{\sqrt{2\omega(\boldsymbol{k})}} \{b(\boldsymbol{k}) \exp[i(kx)] + \bar{b}(\boldsymbol{k}) \exp[-i(kx)]\} ;$$

ove $k_4/i = \omega(\boldsymbol{k}) \equiv +\sqrt{|\boldsymbol{k}|^2 + \mu^2}$, il soprassegno indica il coniugato hermitiano e l'asterisco il coniugato hermitiano per $\nu = 1, 2, 3$ e quello antihermitiano per $\nu = 4$.

Consideriamo il termine dello sviluppo per cui k_ν ha componenti $(0, 0, 0, i\mu)$. La (1) fornisce allora:

$$(3) \qquad \begin{cases} (a_4 - b)|\rangle = 0 ; \\ (a_4^* + \bar{b})|\rangle = 0 . \end{cases}$$

Il più generale ket relativo alla componente di Fourier in questione si può scrivere:

$$(4) \qquad |1, 2, 3\rangle \sum_{N_4', N_b'} c(N_4', N_b') |N_4', N_b'\rangle ,$$

ove il significato di $|1, 2, 3\rangle$ è evidente e N_4', N_b' sono i numeri di bosoni temporali e scalari.

Sostituendo (4) in (3) e tenendo conto che a_4 è un operatore di emissione e $-a_4^*$ un operatore di assorbimento, si ha:

$$\begin{cases} \sqrt{N_4'}\, c(N_4' - 1, N_b' - 1) - \sqrt{N_b'}\, c(N_4', N_b') = 0 ; \\ \sqrt{N_b'}\, c(N_4' - 1, N_b' - 1) - \sqrt{N_4'}\, c(N_4', N_b') = 0 . \end{cases}$$

Da questo sistema si trae immediatamente $N_4' = N_b'$ e $c(N_4', N_b') = \text{cost}$, dimodochè la norma di (4), che è data da $\sum_{N'} |c(N', N')|^2$ è ovviamente infinita.

Ci si convince immediatamente che un'identica conclusione vale anche se si assumesse la (2) in luogo della (1).

(6) Una considerazione analoga vale anche in elettrodinamica.

Introduciamo ora l'operatore metrico η $(=\bar{\eta})$; A_1, A_2, A_3, B si potranno prendere hermitiani e autoaggiunti (secondo la metrica indefinita), A_4 hermitiano ma antiautoaggiunto: introducendo il simbolo † per indicare l'aggiunto, si ha cioè:

$$\begin{aligned} \bar{A}_j &= A_j = A_j^\dagger\,; \\ \bar{A}_4 &= A_4 = -A_4^\dagger\,; \qquad\qquad (A_\mu^\dagger \equiv \eta^{-1}\bar{A}_\mu\eta) \\ \bar{B} &= B = B^\dagger\,. \qquad\qquad (B^\dagger \equiv \eta^{-1}\bar{B}\eta)\,. \end{aligned}$$

È facile vedere che la rappresentativa di η nei numeri di occupazione deve essere tale che:

$$\langle N_4' | \eta | N_4'' \rangle = \delta_{N_4' N_4''}(-1)^{N_4'}\,.$$

Come condizione supplementare assumiamo la (2):

$$\left(\frac{\partial A_\nu^{(+)}}{\partial x_\nu} + \mu B^{(+)}\right)|\rangle = 0\,, \tag{2}$$

che fornisce, per la già considerata componente di Fourier, la prima delle (3):

$$(a_4 - b)|\rangle = 0\,, \tag{3_1}$$

ove però ora sia a_4 che b sono operatori di *assorbimento*. Così facendo, il vettore di stato è normalizzabile, come è facile vedere, e inoltre per gli stati fisici, che soddisfano la nuova condizione supplementare, i gradi di libertà 4 e b non contribuiscono, per la componente di Fourier in questione, all'energia del sistema, come deve essere.

Se facciamo interagire il campo di Stückelberg con quello di Dirac, avremo una equazione di Tomonaga-Schwinger:

$$\mathcal{H}(z)|\sigma\rangle = i\,\frac{\delta|\sigma\rangle}{\delta\sigma(z)}\,, \tag{5}$$

con

$$\mathcal{H} = j_\nu A_\nu + \frac{1}{\mu}\,j_\nu\,\frac{\partial B}{\partial x_\nu} + \frac{1}{2\mu^2}\,(n_\nu j_\nu)^2\,; \tag{6}$$

la (2), a differenza di quanto accade in elettrodinamica, conserva nella descrizione d'interazione la stessa forma che ha nel caso libero.

È ben noto [7] che, mediante una opportuna trasformazione unitaria (la

[7] Vedi, per esempio, il lavoro di GLAUBER citato in [4].

quale non altera le regole di commutazione degli A_ν) la (5) può trasformarsi come segue:

$$(5') \qquad \left(j_\nu(z)A_\nu(z) + \frac{1}{2\mu^2}(n_\nu j_\nu(z))^2\right)|\sigma\rangle' = i\frac{\delta|\sigma\rangle'}{\delta\sigma(z)},$$

mentre la (2) diviene;

$$(2') \qquad \left(\frac{\partial A_\nu^{(+)}(x)}{\partial x_\nu} + \mu B^{(+)}(x) + \int \Delta^{(+)}(x-x')j_\nu(x')\,\mathrm{d}\sigma'_\nu\right)|\sigma\rangle' = 0;$$

per la teoria della matrice S interessa essenzialmente dimostrare il cosiddetto « teorema di Dyson » [8], il quale, nello spazio k, suona:

$$(7) \qquad \left\{\begin{aligned} &K_{\mu\nu}(\boldsymbol{k}, -\boldsymbol{k})\ {}_0\langle\eta\bar{a}_\mu(\boldsymbol{k})a_\nu(\boldsymbol{k})\rangle_0 = 0\,; \\ &K_{\mu\nu}(\boldsymbol{k}, -\boldsymbol{k})\ {}_0\langle\eta a_\mu(\boldsymbol{k})\bar{a}_\nu(\boldsymbol{k})\rangle_0 = K_{\alpha\alpha}(\boldsymbol{k}, -\boldsymbol{k})\,; \end{aligned}\right.$$

ove i $K_{\mu\nu}$ soddisfano alle

$$(8) \qquad \left\{\begin{aligned} &K_{\mu\nu}k_\nu = 0\,; \\ &K_{\mu\nu}k_\mu = 0\,; \end{aligned}\right.$$

e lo stato di vuoto $\rangle_0$ *va definito come lo stato in cui sono assenti i bosoni* (*quadridimensionalmente*) *trasversali a* k_ν; tale stato contiene, compatibilmente colla condizione supplementare, un numero indefinito di bosoni longitudinali e scalari. È, ovviamente, sufficiente provare le (7) per $k_\nu \equiv (0, 0, 0, i\mu)$.

Dalle (8) consegue che $K_{\mu 4}$ e $K_{4\nu}$ sono nulli. Pei valori di aspettazione di vuoto si ha:

$$\begin{aligned} &{}_0\langle\eta\bar{a}_r a_s\rangle_0 = 0\,; \\ &{}_0\langle\eta a_r \bar{a}_s\rangle_0 = \delta_{rs}\,. \end{aligned} \qquad (r, s = 1, 2, 3)$$

Ma ciò è senz'altro sufficiente a dimostrare le (7), in quanto i valori di aspettazione di vuoto che contengono un indice 4 sono indeterminati, ma *finiti*.

Osserviamo infine che, come corollario immediato delle cose dette, si ha che la teoria di matrice S del campo di Stückelberg fornisce gli stessi risultati di una teoria di matrice S del campo di Proca $\varphi_\lambda(x)$ *nella quale si prendano*

([8]) W. PAULI: *Ausgewählte Kapitel aus der Feldquantisierung* (Zürich, 1951), p. 56 e segg.

commutatori dati dalle $[\varphi_\lambda(x), \varphi_\nu(x')] = i\delta_{\lambda\nu}\Delta(x-x')$ *e si dimentichi completamente la* $(\partial\varphi_\lambda/\partial x_\lambda) = 0$.

3. – Consideriamo ora brevemente il caso del campo di Stückelberg *complesso,* in assenza d'interazioni.

I commutatori sono:

$$[A_\lambda(x), A_\nu^*(x')] = i\,\delta_{\lambda\nu}\,\Delta(x-x')\;;$$

$$[B(x), B^*(x')] \;= i\,\Delta(x-x')\;;$$

e tutti gli altri uguali a zero.

La condizione supplementare (1):

$$\left(\frac{\partial A_\nu}{\partial x_\nu} + \mu B\right)|\rangle = 0\,, \tag{1}$$

porta qui di conseguenza:

$$\langle|\left(\mu\bar{B} + \frac{\partial A_\nu^*}{\partial x_\nu}\right) = 0\,. \tag{1''}$$

È immediato convincersi che, a differenza di quanto accade pel campo reale, la (1) e la (1″) *non* danno luogo a incompatibilità banali colle relazioni di commutazione.

Mediante gli sviluppi di Fourier:

$$A_\nu(x) = \frac{1}{\sqrt{V}}\sum_{\boldsymbol{k}}\frac{1}{\sqrt{2\omega(\boldsymbol{k})}}\{a_\nu(\boldsymbol{k})\exp[i(kx)] + \alpha_\nu^*(\boldsymbol{k})\exp[-i(kx)]\}\;;$$

$$B(x) \;= \frac{1}{\sqrt{V}}\sum_{\boldsymbol{k}}\frac{1}{\sqrt{2\omega(\boldsymbol{k})}}\{(b(\boldsymbol{k})\exp[i(kx)] + \bar{\beta}(\boldsymbol{k})\exp[-i(kx)]\}\,,$$

con ragionamenti analoghi a quelli fatti nel § 2 si mostra che da (1) consegue:

$$(b - a_4)\,|\rangle = 0\;;$$

$$(\bar{\beta} + \alpha_4^*)\,|\rangle = 0\,,$$

le quali conducono ancora a un vettore di stato di norma infinita.

È però facile vedere che per evitare questa difficoltà basta introdurre una metrica indefinita, nella quale la rappresentativa, nei numeri di occupazione, di η sia tale che:

$$\langle N_4'^{(a)}N_4'^{(\alpha)}|\,\eta\,|N_4''^{(a)}N_4''^{(\alpha)}\rangle = \delta_{N_4'^{(a)}N_4''^{(a)}}\,\delta_{N_4'^{(\alpha)}N_4''^{(\alpha)}}(-1)^{N_4'^{(a)}+N_4'^{(\alpha)}}\,,$$

ove le notazioni hanno un significato evidente.

La condizione supplementare rimane la

$$\left(\frac{\partial A_\nu}{\partial x_\nu} + \mu B\right)|\rangle = 0 \,.$$

Cioè: a differenza di quanto si ha nel caso reale (e nel caso del campo elettromagnetico) la condizione supplementare resta formalmente inalterata, ma ovviamente il significato degli operatori che in essa figurano è diverso.

Osserviamo, per concludere, che l'introduzione della metrica indefinita sistema completamente la teoria del campo di Stückelberg e, presumibilmente, anche teorie in cui figurino condizioni supplementari, che siano lineari nelle variabili di campo.

* * *

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SUMMARY

The conventional supplementary condition of the Stückelberg field leads to difficulties similar to those given in electrodynamics by the Fermi condition. With the introduction of an indefinite metric such difficulties are completely avoided. The behaviour of the complex field is remarkable: the supplementary condition remains formally unchanged.

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Polarization Effects in Neutron-Proton Scattering at 98 MeV.

P. Hillman and G. H. Stafford

Atomic Energy Research Establishment - Harwell, Berkshire, England

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Summary. — The polarization in free neutron-proton scattering has been measured at 98 ± 3 MeV as a function of scattering angle. A maximum asymmetry $e = 4\%$ corresponding to a polarization of 39% is indicated. The angular distribution departs strongly from anti-symmetry about 90 degrees centre of mass angle.

1. – Introduction.

The angular distribution of the polarization in free proton-proton scattering is known quantitatively at several energies (1). Highly polarized neutron beams are not available, however, so knowledge of the free neutron-proton polarization is less advanced. Wouters (2) at 150 MeV and Dickson and Salter (3) at 110 MeV in early experiments have reported small values of the asymmetry, but did not know the polarization of their beams. Significant results on the variation with angle were obtained by Roberts, Tinlot and Hafner (4) at 150 MeV and by Siegel, Hartzler and Love (5) at 350 MeV.

In addition Marshall, Marshall, Nagle and Skolnik (6) at 314 MeV

(1) See for example, *Proceedings of the Fifth Annual Rochester Conference on High Energy Physics* (New York, 1955).

(2) L. F. Wouters: *Phys. Rev.*, **84**, 1069 (1951).

(3) J. M. Dickson and D. C. Salter: *Proc. Phys. Soc.*, A **66**, 721 (1953).

(4) A. Roberts, J. Tinlot and E. M. Hafner: *Phys. Rev.*, **95**, 1099 (1954).

(5) R. T. Siegel, A. J. Hartzler and W. A. Love: *NYO-7105, Carnegie Institute of Technology*, 1955 (unpublished).

(6) J. Marshall, L. Marshall, D. Nagle and W. Skolnik: *Phys. Rev.*, **95**, 1020 (1954).

and CHAMBERLAIN, DONALDSON, SEGRÈ, TRIPP, WIEGAND and YPSILANTIS [7] at 312 MeV have used highly polarized proton beams to measure the polarization in quasi-free scattering off the neutron in deuterium, mostly counting neutrons and protons in coincidence.

We have measured the angular distribution of the free neutron-proton polarization at an effective energy of 98 ± 3 MeV. The polarization of our neutron beam was $9.8\% \pm 1.4\%$, as determined by WILSON and VOSS [8] in a separate experiment involving small-angle scattering from uranium (SCHWINGER [9]).

2. – Method.

If $I(\theta, \varphi)$ is the intensity for scattering of a polarized beam at centre of mass angle θ measured at an azimuthal angle φ with respect to the plane perpendicular to the direction of polarization, then it may be shown [10] that the asymmetry is:

$$e = \frac{I(\theta, 0) - I(\theta, \pi)}{I(\theta, 0) + I(\theta, \pi)} = P_1 \, P_2$$

where P_1 is the polarization of the incident neutron beam and P_2 is the polarization in the neutron-proton scattering process. The neutrons were produced by bombardment of beryllium targets with the internal proton beam of the 110 inch Harwell synchrocyclotron.

As P_1 was only 9.8% the asymmetries that had to be measured were very small. From a careful consideration of the factors likely to introduce false asymmetries it was decided to measure the relative scattering intensities $I(\theta, 0)$ and $I(\theta, \pi)$ by rotating the counter telescope about the line of the neutron beam. Proton recoils are counted with a triple coincidence telescope. An additional counter in anti-coincidence reduced the background and fixed the upper energy limit.

3. – Apparatus.

The geometrical layout is shown approximately to scale in Fig. 1.

The rotatable table was a rigid duralumin structure four feet long on four inch diameter hollow bearings. The counter telescope was mounted on a

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(7) O. CHAMBERLAIN, R. DONALDSON, E. SEGRÈ, R. TRIPP, C. WIEGAND and T. YPSILANTIS: *Phys. Rev.*, **95**, 850 (1954).

(8) R. WILSON and R. G. VOSS: To be published.

(9) J. SCHWINGER: *Phys. Rev.*, **73**, 407 (1948).

(10) J. V. LEPORE: *Phys. Rev.*, **79**, 137 (1950).

movable arm which was clamped to the table. Each plastic scintillator was mounted in a perspex light guide in a rigid metal frame and had inscribed at its centre a small cross for alignment. The defining crystal was 1.5 cm $\times$ 6.0 cm and 0.3 cm thick a placed at distance of 51 cm from the scatterers.

Three beryllium targets were used in the experiment. They provided neutron beams at plus and minus 26 degrees to the direction of the incident protons and an unpolarized beam at zero degrees. The neutrons were collimated by a 1.625 inch $\times$ $\times$ 0.625 inch wave guide 54 inches long followed by a slightly larger screening collimator 18 inches long. The collimator was set in a concrete shielding wall 6 feet thick.

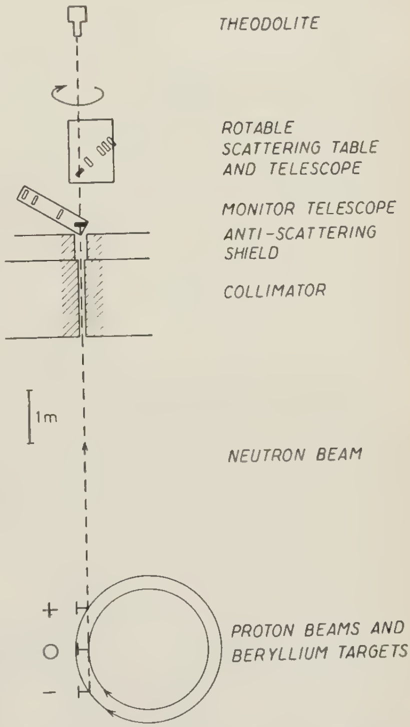

Fig. 1. – Experimental Geometry, approximately to scale.

The second scatterers were polythene and carbon of equal stopping power. The counting geometry was defined by the scatterer and the second scintillator. Each scatterer was mounted within a duralumin framework by two 0.008 inch diameter wires passing at right angles through it. It presented a 1 cm $\times$ 3.8 cm face to the counter telescope and had a thickness which was changed with angle in order to preserve the same effective neutron spectrum. Both the rotable table and the scatterer changer were remotely controlled.

The monitor used was a triple coincidence proton recoil telescope. It was similar to the counter telescope so that the effect of small cyclotron energy variations was largely cancelled out. Proton recoils originated in a polythene radiator 1 cm thick placed immediately after the collimator. A boron trifluoride slow neutron counter near the apparatus was also used as a monitor throughout the experiment as an additional check on the behaviour of the equipment.

4. – Procedure.

Great care was taken to ensure that no false asymmetry was introduced by the incorrect alignment of the apparatus. The procedure that we adopted was as follows. Cross wires were placed in both ends of the collimator and a

theodolite aligned with them. The three beryllium targets, the cross wires in the bearings of the rotatable table and finally the polythene and carbon scatterers were aligned with the cross hairs of the theodolite. Whenever possible the alignment was also checked at the end of a run. Our measurements extended over a period of 6 months and during this time the targets and the rest of the apparatus were aligned many times. We have estimated that it was possible, with the theodolite, to position the targets to an accuracy of ± 0.010 inches, the table to ± 0.003 inches and the scatterers to ± 0.003 inches.

Frequent short runs were taken in a regular sequence to reduce the effect of small changes in cyclotron operating conditions which might have affected the neutron spectrum and of changes due to variations in amplifier gain and photomultiplier E.H.T. voltage. No changes in the cyclotron controls were made throughout an asymmetry measurement cycle. As will be discussed in section 6 below the use of targets producing polarized neutrons by scattering at plus and at minus 26 degrees enabled certain false asymmetries to be cancelled if at each second scattering angle results of equal statistical weight were obtained on both targets. Owing to the occasional failure of the minus 26 degree target which was situated within the dee this was not always possible. However, the false asymmetries can be calculated with sufficient accuracy to make the necessary correction small.

Measurements were also made with a zero degree target as this provided a check on the satisfactory alignment of the table and counter telescope with the beam axis, and on the magnetic field effects, as discussed below (§ **6**).

5. – Neutron Energy.

Our counter geometry and scatterer thicknesses were such that the neutron spectrum was approximately triangular extending from 79 to 121 MeV. The effective neutron energy was 98 ± 3 MeV as calculated from a measurement of the attenuation of the beam by polythene, using the results of TAYLOR, PICKAVANCE, CASSELS and RANDLE [11]. This was kept constant at different angles by using scatterers and aluminium absorbers of appropriate stopping power. To improve the counting rate at 60° and 70° c.m., however, the scatterer thickness was increased. At these angles, the effective energy was therefore lowered by 5 and 3 MeV respectively.

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(11) A. E. TAYLOR, T. G. PICKAVANCE, J. M. CASSELS and T. C. RANDLE: *Phil. Mag.*, **42**, 328 (1951).

6. – Corrections and Errors.

False asymmetries can arise from a number of different sources.

6·1. *Geometrical effects.* – At a first scattering angle of 26° the neutron intensity changed by about 6% per degree (SNOWDEN [12]). Any non-rigidity of the table on rotation, which would have shifted the second scatterer, must therefore be avoided. The rigidity was tested by looking with a theodolite at cross wires in the bearings of the table. There was no detectable horizontal movement of the front cross wire and less than 0.002 inches of the rear cross wire when the table was rotated through 180 degrees. This corresponded to a negligibly small asymmetry.

However, even with perfect geometry the non uniform irradiation of the scatterer produced a change in the effective scattering angle on rotation. The counting rate varied by as much as 20% per degree second scattering angle because of variation in scattering cross-section and because of the change in the effective neutron threshold for a telescope of fixed stopping power.

The false asymmetry corresponding to this effect varied with angle and had a calculated maximum value of 0.02%. It was not cancelled by the use of plus and minus targets.

The geometrical variation in the scattering angle on rotation of the table was checked by mounting a mirror with its reflecting surface in the scatterer position and viewing the reflected image of the cross inscribed on one of the scintillators with a theodolite that had been aligned along the axis of rotation. When the table was rotated the angular motion measured was less than 0.5 minute. This included the error in positioning the mirror along the axis of the beam. This corresponded to a false asymmetry of 0.1% at a centre of mass scattering angle of 60° and 0.03% at a scattering angle of 160°.

Misalignments of the cyclotron targets, the scatterers or the table also could have produced false asymmetries, in conjunction with the beam non-uniformity and counting rate change with angle. Several independent alignments were made at each angle. We have placed an upper limit of $\pm 0.1\%$ on the false asymmetry arising from these sources.

6·2. *Magnetic effects.* – At the position of the table the residual cyclotron field was 8.2 ± 0.5 gauss. This affected the measurements in two ways.

(*a*) It changed the effective scattering angle in the left and right positions by bending the paths of the recoil protons. The calculated asymmetry

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([12]) M. SNOWDEN: *Phil. Mag.*, **43**, 285 (1952).

produced in this way varied from 0.1% at 160° centre of mass angle to 0.8% at 60°. The corrections made to our measurements were, however, small or zero because the effect was of opposite sign for the plus and minus 26° targets.

(*b*) In going from left to right the orientation of the counters changed with respect to the direction of the magnetic field and although the counters were shielded with steel and mu-metal cylinders there was a possible change in counting efficiency. The value $e = +0.02 \pm 0.29\%$ of the average of all the measurements made with the zero degree target set an upper limit on this effect.

6'3. *Background Correction.* – At the small scattering angles the background counting rate differed considerably from left to right and in the worst case was 40% of the polythene counting rate. The conventional method of background subtraction tends to overcorrect owing to the shadowing effect when the scatterer is in position and if the background is not the same on both sides this introduces a false asymmetry. This effect was not cancelled by the use of plus and minus targets. We have estimated that the correction varied from approximately 0.1% at 60° scattering angle to zero at 160°.

7. – Results.

The asymmetries for hydrogen are given in Table I. The errors quoted are standard deviations. Most of the error is due to counting statistics. All corrections discussed in section 6 have been made. We have also included the values of e obtained for carbon. Errors are large because counting times were chosen to give the best statistics for hydrogen.

TABLE I.

Laboratory Scattering angle of proton (degrees)	Asymmetry e (percent)	
	Hydrogen	Carbon
10	$+0.1 \pm 0.7$	$+0.2 \pm 1.4$
15	$+0.7 \pm 0.6$	$+1.1 \pm 1.2$
20	$+0.8 \pm 0.5$	$+2.8 \pm 0.8$
30	$+0.4 \pm 0.8$	$+0.8 \pm 1.1$
40	-1.7 ± 1.0	$+2.2 \pm 1.2$
45	-2.1 ± 1.4	$+3.6 \pm 2.1$
50	-3.5 ± 1.2	$+0.6 \pm 2.0$
55	-4.0 ± 1.3	-1.8 ± 2.2
60	-2.8 ± 1.7	0.0 ± 2.5

In Fig. 2 the results are plotted in the form of $P(\mathrm{d}\sigma/\mathrm{d}\Omega)$. The values of $\mathrm{d}\sigma/\mathrm{d}\Omega$ (the unpolarized differential (n, p) cross-section) were obtained by interpolation between the results of STAHL and RAMSEY [13] at 91 MeV and those of RANDLE and SKYRME [14] at 133 MeV. The absolute scale of P is uncertain by the experimental error of $\pm 14\%$ in the polarization of the neutron beam.

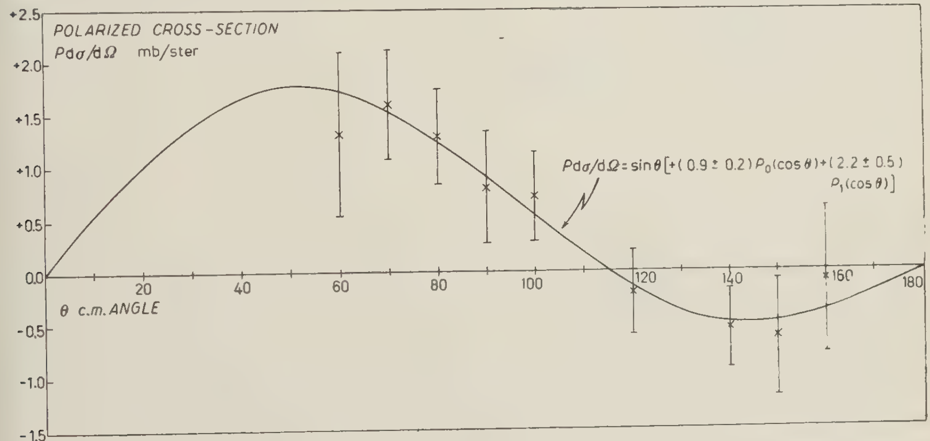

Fig. 2. – Free neutron-proton polarization at 98 MeV. The curve is a least squares fit.

8. – Discussion.

From the results of WILSON and VOSS' experiment [8] it is known that our neutron beam from the $+26°$ target was polarized with spin up. With the convention that spin up scattering to the left corresponds to positive polarization, our results are positive at small scattering angles. This is the same sign as that found in proton-proton and proton-nuclear scattering. (BRINKWORTH and ROSE [15], MARSHALL and MARSHALL [16]).

The polarization cross-section should have the form [17]

$$P\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sin\theta[a_0P_0(\cos\theta) + a_1P_1(\cos\theta) + a_2P_2(\cos\theta) + \ldots]\,.$$

From Klein's phase shift analysis [17] it appears that all terms other than the first two may be small. The curve drawn in Fig. 2 is a least squares fit

(13) R. H. STAHL and N. F. RAMSEY: *Phys. Rev.*, **96**, 1310 (1954).
(14) T. C. RANDLE and D. SKYRME: Private communication (1955).
(15) M. BRINKWORTH and B. ROSE: *Nuovo Cimento*, **3**, 195 (1956).
(16) L. MARSHALL and J. MARSHALL: *Phys. Rev.*, **98**, 1398 (1955).
(17) C. A. KLEIN: *Nuovo Cimento*, **2**, 38 (1955).

to the experimental points using the first two terms only:

$$P\,\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sin\theta[(0.9 \pm 0.2)\,P_0(\cos\theta) + (2.2 \pm 0.5)\,P_1(\cos\theta)]\,.$$

If the theoretical curve is a significant one the fit is rather better than could reasonably be expected statistically. However, we feel certain there was no experimental bias.

Using three terms the best fit is given by

$$P\,\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \sin\theta[0.8 \pm 0.3)\,P_0(\cos\theta) + (2.0 \pm 0.7)\,P_1(\cos\theta) - (0.4 \pm 0.8)\,P_2(\cos\theta)]$$

which differs only slightly from the two-term fit.

If the Serber « half-ordinary, half-exchange » force were valid for n-p scattering at 100 MeV, as suggested by the near symmetry of the unpolarized angular distribution about 90°, the polarized cross-section would be antisymmetrical about 90° (WOLFENSTEIN (18); SWANSON (19)) and have the form

$$P\,\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = a_1 \sin\theta\,P_1(\cos\theta)\,.$$

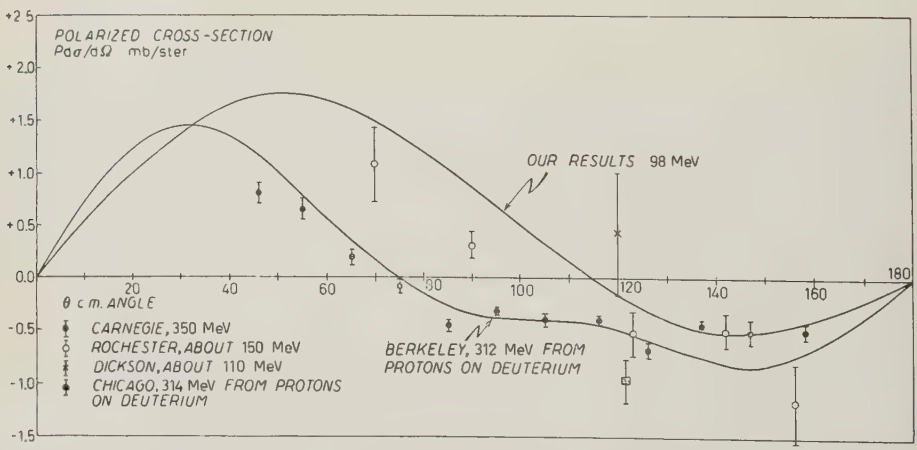

Fig. 3. – Free neutron-proton polarization at various energies. The « Harwell » curve is the least squares fit to the present results. The « Berkeley » curve is a least squares fit to the results of CHAMBERLAIN *et al.* (7). The « Rochester », « Dickson », « Chicago » and « Carnegie » points are those of ROBERTS *et al.* (4), DICKSON and SALTER (3), MARSHALL *et al.* (6) and SIEGEL *et al.* (5).

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(18) L. WOLFENSTEIN: *Phys. Rev.*, **76**, 541 (1949); **82**, 308 (1951).
(19) D. R. SWANSON: *Phys. Rev.*, **84**, 1068 (1951); **89**, 749 (1953).

The probability that out results conform to such a distribution is less than 1%. It is concluded that there is at least one appreciable p phase shift present at 100 MeV.

A comparison with the results of other investigations is made in Fig. 3. In plotting the Rochester results we have used a value of 15% for the polarization of their beam as suggested by WILSON [8] and values of $d\sigma/d\Omega$ calculated from the results of RANDLE and SKYRME [14]. The Chicago point uses the values of $d\sigma/d\Omega$ of KELLEY, LEITH, SEGRÈ and WIEGAND [20].

There is no significant change in the magnitude of the polarization with energy but a trend in the angular distribution appears to be discernible.

* * *

We wish to thank the cyclotron operating crew for cheerful cooperation at all times.

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(20) E. KELLY, C. LEITH, E. SEGRÈ and C. WIEGAND: *Phys. Rev.*, **79**, 96 (1950).

RIASSUNTO (*)

La polarizzazione nello scattering neutrone-protone libero è stata misurata a 98 ± 3 MeV in funzione dell'angolo di scattering. Si è riscontrata un'asimmetria massima $e=4\%$ corrispondente a una polarizzazione del 39%. La distribuzione angolare si scosta fortemente dall'antisimmetria di un angolo di circa 90° rispetto al centro di massa.

(*) *Traduzione a cura della Redazione.*

NOTE TECNICHE

A Simple Threefold Concidence Circuit Using Only one EQ 80 (6 BE 7) Valve.

E. Pohl

Istituto di Fisica dell'Università - Padova
Istituto Nazionale di Fisica Nucleare - Sezione di Padova

(ricevuto il 25 Gennaio 1956)

Summary. – The three control grids of an EQ 80 which are each separated by screen grids, when polarized to mantain the valve normally cut off, serve as inputs for the channel of a triple coincidence. The circuit is completely insensitive to single and double coincidences, and is economical both in valves and current.

In cosmic-ray experiments threefold coincidences are often necessary. This is usually done by means of the Rossi-circuit, using three pentodes which are normally conducting. In experiments using several threefold coincidence steps it is more economical both in valves and current to use multi-grid tubes, as first utilized by Bothe for twofold coincidences: here one makes use of a valve with 2 control-grids, such as a hexode mixer. Both control-grids have strong negative bias so that the valve remains cut-off if a positive pulse is applied to only one grid; however, when positive pulses are applied to both grids simultaneously a large negative pulse will appear at the anode resistor. This principle can be extended to the case of threefold coincidences, e.g. using pentodes which have separate suppressor grid connections, such as the EF 80 (6 BX 6) which we have operated with success. The drawback of such a tube lies in the markedly different characteristics of the three grids; in particular the suppressor grid requires a large negative bias to cut-off the valve, while the screen grid needs extremely large positive pulses

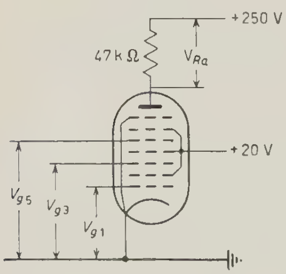

Fig. 1. – Circuit for the determination of the characteristics of the EQ80 valve given in Fig. 2.

to switch it on. This need for large pulses and the relative large capacities between the electrodes favours coupling.

These difficulties can be avoided by the use of the Philips Enneode EQ 80

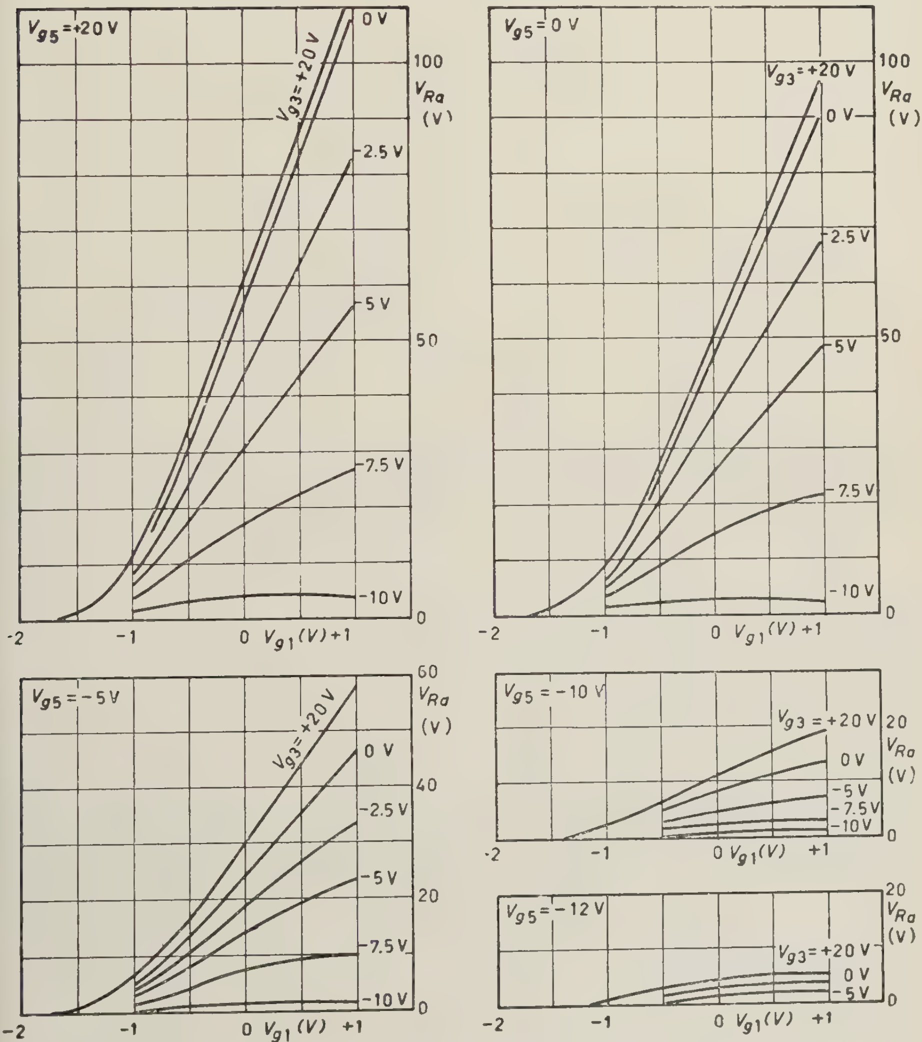

Fig. 2. – Characteristics of the EQ80 tube when used as a coincidence valve.

(6 BE 7) which has been developed as an F.M. detector and limiter and as an A.F. amplifier. The three control grids g_1, g_3, g_5 are separated from each other and from the suppressor grid by the internally connected screen grids g_2, g_4, g_6. Fig. 1 shows the valve in the circuit we have used for determining the anode characteristics as a function of the three grid voltages, information so far unpublished. With the anode resistor of 47 kΩ, one obtains output

pulses up to 100 V having rise times which permit resolving times down to two μs.

Fig. 2 shows the characteristics so obtained. In the diagrams V_{Ra} is plotted against V_{g1}, with V_{g3} as parameter, each diagram being repeated for five different values of V_{g5}.

The minimum bias whereby any one grid will maintain the valve cut-off, irrespective of what happens at the other two grids, may be obtained from the diagrams; these values are $V_{g1} \cong -2$ V, V_{g3} -12 V, $V_{g5} \cong -16$ V. To allow for variations between tubes, the following are safe values -5 V for g_1, -15 V for g_3 and -18 V for g_5. Under these conditions, the circuit is completely insensitive to single and double coincidences.

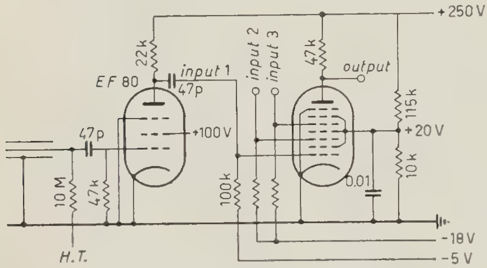

Fig. 3. – Simple threefold coincidence circuit for use with relatively large Geiger counters. The preamplifier is repeated for each of the three input grids.

For an output pulse of 80 to 90 V in a threefold coincidence the three grids must be driven respectively to $+1$ V, 0 V, and 0 V, corresponding to pulse amplitudes of about 6 V, 15 V, and 18 V. By the use of larger pulses, a maximum output pulse of 100 V may be obtained, this limit being set by grid currents. A suitable increase in the biases can be used to eliminate small, undesirable, input pulses.

Fig. 3 shows a simple, complete coincidence circuit with pentodes as preamplifiers, and having a resolving time of 5 to 6 μs. If the resolving time is required to be constant and independent of the shape and size of the input pulses, a suitable pulse-shaper should be used, as is standard practice with other coincidence circuits.

It is clear that one may combine coincidences and anticoincidences with the EQ 80 valve. A negative anticoincidence pulse may be fed in through g_3 or g_5, the grid concerned being polarized to $+20$ V (g_1 is unsuitable since its use would result in large grid and screen-grid currents), while, simultaneously, coincidence pulses may be fed in through the other grids. Thus, with a single valve, one may make a double coincidence with an anticoincidence.

RIASSUNTO

Le tre griglie controllo di una EQ 80, separate da griglie schermo, fungono da entrate per i canali di una coincidenza tripla, e sono polarizzate in modo che la valvola normalmente non conduce. La risposta a coincidenze singole e doppie è completamente nulla. L'impiego di questo circuito offre un risparmio in numero di valvole e in corrente anodica.

Absorption of K^--Particles by Nuclei (*).

W. B. CHESTON

University of Minnesota - Minneapolis, Minnesota

(ricevuto il 21 Dicembre 1955)

It is known [1] that when a K_2^- particle [2] is absorbed by a nucleus, a nucleon in the absorbing nucleus carries out a transition to a hyperon state via the reaction:

$$(1a)\qquad K_2^-\ \text{(Bohr orbit)} + (A\text{-nucleons}) \to \to (A-1)\ \text{nucleons} + \text{hyperon} + \text{pion}$$

$$(1b)\qquad K_2^-\ \text{(Bohr orbit)} + (A\text{-nucleons}) \to \to (A-1)\ \text{nucleons} + \text{hyperon}\,.$$

The hyperon of eq. (1) may be either a Λ or a Σ. We have estimated the fraction of the time the absorption act should proceed without emission of a real pion considering the coupling of the nucleon absorbing the K_2 particle with the other nucleons of the absorbing nucleus via a static pion field [3]. An interaction Hamiltonian density of the simplest form to describe the absorption of a K_2 particle by a nucleon has been employed namely:

$$(2)\qquad \mathcal{H}^{\text{Int}} = \{\eta_Y(\Psi_N^+\Psi_Y)(\Phi_K^+\varphi_\pi) + \text{h.c.}\}\,,$$

where Ψ_N, Ψ_Y, Φ_K, $\varphi_\pi \equiv$ wave amplitude for the nucleon, hyperon, K-particle and pion fields respectively; $\eta_Y \equiv$ coupling constant describing the «strong» interaction among the four fields. For the lighter elements of a nuclear emulsion (C, N, O), the relative number of absorptions occurring without pion emission is:

$$\frac{(\tau_Y)^{-1}}{(\tau_{Y\pi})^{-1}} \approx \begin{cases} 1.4.\,10^{-3}\,(g^2/\hbar c)A & \text{for } Y \equiv \Sigma \\ 7.6\cdot 10^{-3}\,(g^2/\hbar c)A & \text{for } Y \equiv \Lambda \end{cases}$$

where $g \equiv$ nucleon-pion (PS) coupling constant. The numerical factor preceding the fundamental constant decreases by about 5% when A increases from 12 to 16. For the CNO group, non-mesonic absorption (reaction (1b)) should occur

(*) Supported in part by the joint program of the U.S. Atomic Energy Commission and the U.S. Office Naval Research.

(1) G. GOLDHABER, private communication; W. FRY, private communication; see also, for example, NAUGLE, NEY, FREIER and CHESTON: *Phys. Rev.*, **96**, 1383 (1954). The author is indebted to Drs. GOLDHABER and FRY for making the data of their groups available prior to publication.

(2) We use the notation of R. SACHS: *Phys. Rev.*, **99**, 1573 (1955).

(3) The estimation procedure is similar to that carried out in the case of non-mesonic decay of hyperfragments (see W. CHESTON and H. PRIMAKOFF: *Phys. Rev.*, **92**, 1537 (1953)).

in approximately 20% of the cases when the associated hyperon is a Σ but in approximately 55% of the cases when the produced hyperon is a Λ. These figures mirror qualitatively the trend of the experimental data on K_2^- absorptions [1]. The increase in the relative number of non-mesonic absorptions accompanying an increase in the energy release in the absorption can be understood in the following manner. The wave number of the emitted pion in the absorptions with pion emission (reaction (1*a*)) increases by $\approx 40\%$ when the mass of the emitted hyperon decreases from that of the Σ to that of the Λ (($q_\Sigma \approx 1.24\varkappa_\pi$; $q_\Lambda \approx 1.78\varkappa_\pi$) resulting in a suppression of the coherent effect of the absorbing nucleons. The absorbing nucleons in the absorptions unaccompanied by pion emission (reaction (1*b*)) act incoherently in both the Σ and Λ emission since the wave number of the emitted hyperons is, in both cases, $\gg$ than $\varkappa_\pi$ ($k_\Sigma \approx 5.7\varkappa_\pi$; $k_Y \approx 6.5\varkappa_\pi$). The phase space factors involved are actually less favorable for pion emission in the case of Σ production since $k_\Sigma/q_\Sigma \approx 4.6$ wheareas $k_\Lambda/q_\Lambda \approx 3.7$.

To compare the estimate quoted above with experiment, the absorption by the nucleus of any real pions produced in the K_2 absorption must be taken into account. This will decrease the relative number of absorptions with observed pion emission (correcting for the unobserved neutral pions by charge independence arguments) under that estimated above, and the more energetic pions will be preferentially absorbed. In addition, many of the K_2 absorptions will occur in the Ag and Br in the emulsions for which it is difficult to perform the estimation procedure employed in the case of the CNO group. Finally, any momentum dependence in the interaction Hamiltonian density describing the K_2 absorption will tend to increase the relative number of absorptions without pion emission since $k_Y/q_Y > 1$.

Au sujet du volume sensible des compteurs de Geiger-Müller à cathode externe.

A. M. BAPTISTA et J. P. GALVÃO

Centros de Estudos de Energia Nuclear, Laboratório de Física - Lisboa

(ricevuto il 7 Gennaio 1956)

Dans une récente publication (1) D. BLANC montre, d'accord avec les résultats que nous avons obténus (2), que le volume sensible des compteurs à cathode externe comme celui qui est représenté dans la fig. 1 n'est pas généralement

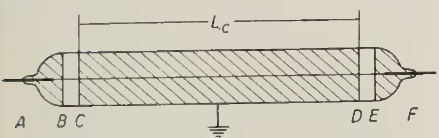

Fig. 1. – Schéma des compteurs utilisés.

coïncident avec le volume limité par la longueur graphitée L_c; placée à la masse, séparée des zones AB et EF au potentiel anodique, par deux zones isolantes BC et DE.

D. BLANC a montré expérimentalement comment varie le volume sensible avec la longueur de la zone isolante, obtenant une relation simple $L=L_c+0.2L_i$ valable pour le seuil de Geiger, où L est la longueur efficace, L_c la longueur graphitée placée à la masse et $L_i=BC+DE$ la longueur totale des zones isolantes. C'est-à-dire, le volume sensible représenté par la longueur éfficace L serait toujours plus grand que la longueur L_c.

Cependant, dans notre publication (2), nous avons montré que la longueur efficace L pourrait être inférieure à L_c pour des valeurs L_i suffisamment petites. Ce fait est mis en évidence dans la fig. 2, où l'on présente la courbe de variation de la vitesse de comptage au long d'une génératrice du compteur obtenue avec une source bien collimée de $^{90}Sr+^{90}Y$.

Nous avons crû intéressant de confirmer ces résultats employant des compteurs à cathode multiple du modèle présenté par D. BLANC (1) où chaque zone graphitée fonctionne indépendemment, si toutes les autres sont placées à la tension du fil anodique.

Dans la fig. 3, nous présentons les résultats obtenus qui confirment nos conclusions antérieures, c'est à dire, une longueur efficace inférieure à la longueur correspondante à la zone graphitée placée à la masse L_c pour des petites valeurs de L_i.

(1) D. BLANC: *Nuovo Cimento*, 1, 1280 (1955).

(2) A. M. BAPTISTA, R. H. CORDEIRO, e J. P. GALVÃO: *Rev. da Fac. de Ciências de Lisboa*. 4, 5, (1955).

Ce manque d'accord nous semble avoir une simple interprétation. En effet, étant R_1 la résistance du verre limitée par la couche isolante extérieure BC, en série avec la résistance r correspondante à la

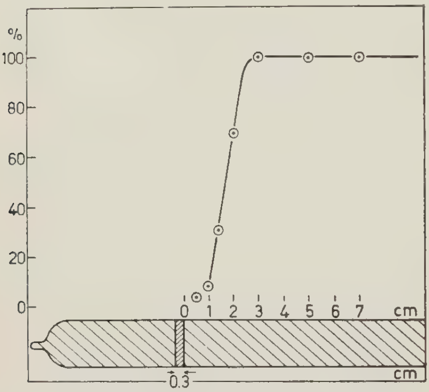

Fig. 2. – Courbe répresentative du volume sensible d'un compteur à cathode externe. En ordonnées, des comptages par unité de temps en unités arbitraires. En abcisses, la distance du faisceau collimé d'une source de $^{90}Sr + {}^{90}Y$ à la ligne de séparation zone cathodique - zone isolante.

partie du verre limitée par la longueur graphitée placée à la masse, si V est la tension appliquée au fil anodique, la

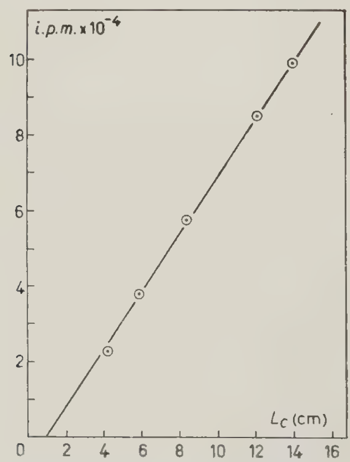

Fig. 3. – Variation du taux de comptage pour le seuil de Geiger en fonction de L_c.

chute de tension le long des résistances r et R_1 sera $V = R_1 i_1 + r i_1$ étant i_1 l'intensité du courent qui les parcourt.

D'autre part le potentiel entre le fil anodique et la parois interne du compteur non irradié croît d'une valeur sensiblement égale à zéro, zone AB, jusqu'à une valeur près de $V - ri_1$, zone cathodique CD, et d'où la possibilité du champ électrique à l'intérieur du compteur dans une parte de la zone BC d'être suffisant au fonctionnement dans la région de Geiger-Müller, et alors le volume sensible correspondant s'alongera à l'intérieur de la zone isolante.

Si, maintenant, nous considérons que la longueur de BC est très petite nous aurons $R_2 < R_1$ et $i_2 > i_1$, et la chute de potentiel $ri_2 > ri_1$ pourra être telle que près de la ligne de séparation — zone isolante-surface cathodique — elle provoque une réduction du champ électrique assez forte pour que le volume sensible puisse être inférieur à la longueur de la surface cathodique CD.

Il faut encore admettre que la limitation du volume sensible des compteurs à cathode externe soit aussi dépendente de l'intensité de la source radioactive, car, comme il est bien connu, le potentiel de la « cathode virtuelle » [2] varie avec l'intensité d'irradiation.

D'accord avec les raisons ci-dessus on voit aussi pourquoi le palier augmente avec la longueur BC, comme ont verifié Blanc et Scherer [3].

Donc nous pouvons conclure que le problème de la limitation du volume sensible dans les compteurs à cathode externe devra toujours être résolu, dans chaque cas, d'accord avec les caractéristiques du tube de verre employé dans leur construction et dépendra encore, en quelque façon, de l'intensité de la source.

(3) D. Blanc et M. Scherer: *C. R. Acad. Sciences*, **228**, 2018 (1949).

A Test of Neutrino-Antineutrino Identity (*).

C. L. Cowan Jr., F. B. Harrison, L. M. Langer (+) and F. Reines

University of California, Los Alamos Scientific Laboratory - Los Alamos, New Mexico

(ricevuto il 19 Gennaio 1956)

Results published by McCarthy ([1]) indicate that ^{48}Ca and ^{96}Zr undergo double beta decay with no net emission of antineutrinos. This would suggest that the hypothesis of Majorana as applied by Furry to double beta decay ([2]), which assumes that the neutrino and antineutrino are identical, is correct. Later measurements by Awschalom ([3]) gave negative results for the same two isotopes.

As the separated isotope ^{150}Nd was available in gram quantities, an experiment was performed using this isotope in the attempt to detect the simultaneous emission of two beta particles. The energy available for this process is 4.4 ± 0.8 MeV plus the rest mass energy of the two beta particles, where the limits quoted may be taken as two standard deviations ([4]). Furthermore, the transition ^{150}Nd to ^{150}Sm by double beta emission is favored by a factor of four over the similar decay of ^{48}Ca and by a factor of eight over the ^{96}Zr decay because of the Coulomb-nuclear size factor ([5]). If no antineutrinos are emitted in the double beta decay process, the electrons should share the available energy, and the sum of their energies should be a constant. A reasonable value for the mean life of ^{150}Nd for double beta decay is 1.3×10^{15} years for a kinetic energy release of 4.4 MeV. Under the most severe assumptions one would not expect this to exceed $6 \cdot 10^{17}$ years.

A 2.142 g sample of Nd_2O_3 made from Nd enriched ([6]) to 88.23% ^{150}Nd was mounted between two layers of 0.9mg/cm² aluminum coated Mylar film. The average thickness of source and film was 30 mg/cm². The mounted source was immersed in a liquid scintillator and viewed on each side from a distance of

(*) Under the auspices of the United States Atomic Energy Commission.

(+) Permanent address: Dep't. of Physics, University of Indiana, Bloomington, Indiana.

([1]) J. A. McCarthy: *Phys. Rev.*, **97**, 1234 (1955); **90**, 853 (1953).

([2]) E. Majorana: *Nuovo Cimento*, **14**, 171 (1937); W. H. Furry: *Phys. Rev.*, **56**, 1184 (1939).

([3]) Private communication, Dr. M. Awschalom.

([4]) Private communication, Dr. B. G. Hogg.

([5]) Private communication, Dr. E. J. Konopinski. Professor Konopinski has discussed certain theoretical questions relative to double β-decay in *Los Alamos Report LAMS* 1949; H. Primakoff: *Phys. Rev.*, **85**, 888 (1952).

([6]) Our samples were furnished by the Isotope Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee.

three inches by a 5-inch Dumont type 6364 photomultiplier tube. The efficiency of the system for detection of both double beta decay electrons, one on each side of the source, was calculated to be 30% for a total kinetic energy release of 4 MeV. A background sample of Nd_2O_3 prepared from Nd depleted (6) to 0.064% ^{150}Nd was mounted similarly acceptance were that each pulse of a pair must lie between 0.35 and 8.0 MeV, and that they must be coincident within 0.5 μs. The energies of such pairs were summed electronically and analyzed with a 100 channel pulse height analyzer. The energy calibration was based on the peaks produced by minimum ionizing cosmic rays passing through the scintillator

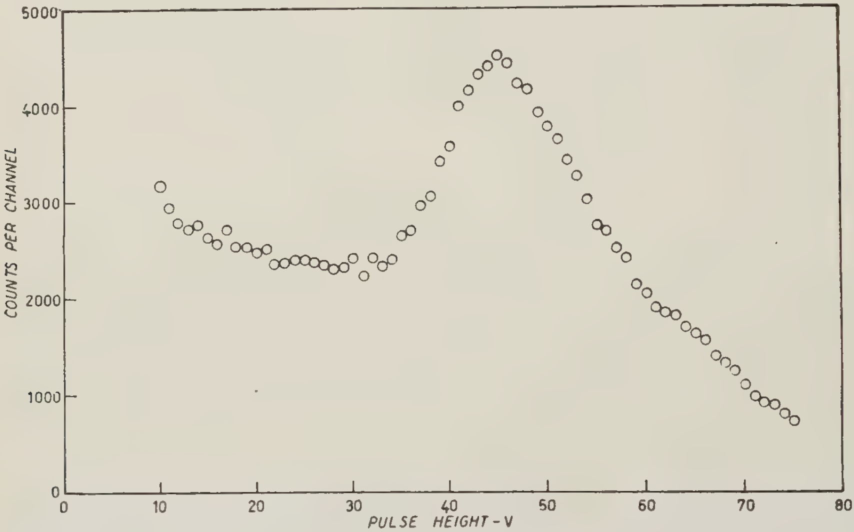

Fig. 1. – Calibration curve obtained using minimum ionizing cosmic ray particles passing through one of the scintillator chambers at the ^{150}Nd sample.

and placed in the scintillator between a second pair of photomultiplier tubes. Light shields were provided, and an axle permitted periodic interchange of the two samples. This system was placed in the center of a 600 liter tank of liquid scintillator viewed by twelve 5-inch photomultiplier tubes operated in parallel, providing an effective anticoincidence shield for charged particle background. The apparatus was surrounded by a minimum of four inches of lead or bismuth on all sides.

In recording the data, pairs of pulses from either pair of photomultiplier tubes viewing the samples were accepted if they were not in coincidence with the guard scintillator. The conditions for (Fig. 1). The width of this peak is due to the geometry and to the Landau effect.

Fig. 2 contains the data obtained with the ^{150}Nd sample. As no positive effect was found outside the statistical fluctuations, the data obtained with the depleted blank are omitted. The residual background is attributed largely to natural radioactivity of the photomultiplier tubes, and to neutrons arising from interactions of the cosmic radiation with the detector shield. A neutron source placed immediately outside the lead shield gave a spectrum of similar shape, with the same peak at 5.6 MeV, which is believed to be due to two successive Compton scatterings of the 7.6 MeV γ-ray from neutron capture in iron.

For our equipment a count rate of 0.10 per hour corresponds to a mean life for ^{150}Nd of $2.2 \cdot 10^{18}$ years. We estimate our energy resolution (full width at half maximum) in the region of interest to be 0.5 MeV, or five channels on the pulse height analyzer. It is seen in Fig. 2 that the deviations of the experimental points from an assumed « best fitting » curve may be attributed solely to statistical fluctuations. The statistical treatment of such negative results is, of course, very much a matter of personal choice. As an aid to such considerations, we state that the integral counting rate in a 0.5 MeV interval associated with one standard deviation from the curve of Fig. 2 is 17 counts per 358 hours, or 0.048 c/hr.

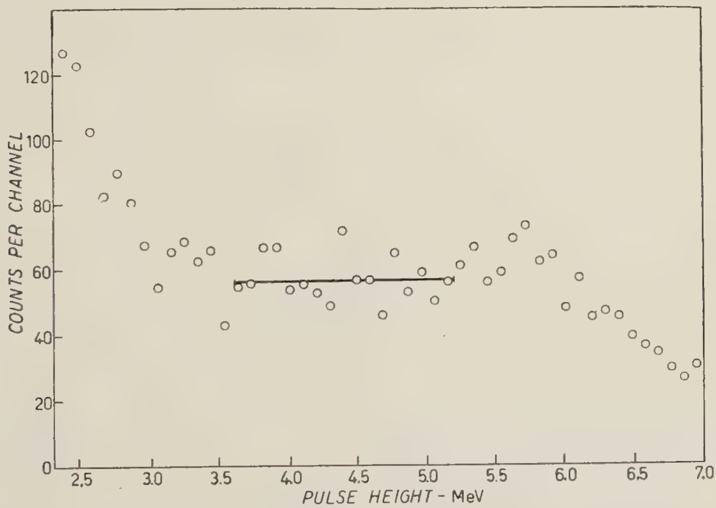

Fig. 2. – Data collected in 358 hours using the ^{150}Nd sample. Each point represents the counts obtained at the energy sum indicated.

We conclude from these data that under present theoretical considerations the Majorana-Furry hypothesis does not apply to the neutrino. This leaves as a plausible alternative the assumption that the neutrino is a Dirac particle.

* * *

We wish to thank Mr. HERALD KRUSE and Mr. MARTIN WARREN for their assistance in the experiment and Dr. A. D. MCGUIRE for interesting discussions of the work.

Mesonic Decay in Flight of a Triton Hyperfragment.

O. Skjeggestad and S. O. Sørensen
Institute of Physics, University of Oslo, Norway

(ricevuto il 23 Gennaio 1956)

A number of experiments with photographic plates exposed to cosmic radiation have established the fact that a Λ^0-particle together with a number of nucleons can form nuclei which are stable for times of the order of the lifetime of the free Λ^0. Among these hyperfragments examples of meson-active tritons $^3H^*$ have been identified. In most of the cases the binding energy of the Λ^0-particle in $^3H^*$ is found to be less than ~ 1 MeV, except in an event described by Yagoda (1) who observes a binding energy of 3.2 ± 1 MeV. The latter observation, however, may possibly be an example of a $^4H^*$ hyperfragment. The study of the decay in flight of light hyperfragments is particularly advantageous because it often permits an unambiguous identification of the various particles involved in the decay process. An event representing the decay in flight of an unstable $^3H^*$ nucleus has recently been observed in our laboratory. It was recorded in a stack of stripped Ilford G-5 emulsions, each 600 μm thick, exposed at high altitude by means of free balloons. A projection drawing of the event is shown in Fig. 1.

From a nuclear disintegration of type 23+3p a particle A is emitted which in S produces a three prong star B, C, D. The track A is ~ 44 μm, and has the appearance of a particle still in flight when producing the secondary star. The tracks A and B are colinear within the errors of measurements. B is brought to rest in the stack of plates and has a range of 1578 μm, showing the characteristic appearance of a heavy particle of unit charge. The prong C is due to a negative π-meson which stops after traversing 9013 μm of emulsion and produces a σ-star. The track of this meson is very flat, and in estimating its energy corrections have been made for the ~ 6000 μm which the particle traverses in the tissue paper spacers between the strips of emulsion. In calculating these corrections we have estimated the stopping power of photographic emulsion to be ~ 12 times that of the tissue paper, for π-mesons in the energy region considered. Using the range-energy relations cal-

(1) H. Yagoda: *Phys. Rev.*, **98**, 153 (1955).

culated by BARONI *et al.* (2), the kinetic energy of the π-meson is 22.80 MeV. The track D also stops in the emulsion with a range 2958 μm, and is produced by a heavy particle of unit charge. The angles in space between the three prongs the plane of the emulsion and the three tracks B, C and D being respectively 52.2° (up), 14.0° (down) and 40.0° (up).

The general appearance of the event strongly indicates that the particle A is a light hyperfragment of unit charge con-

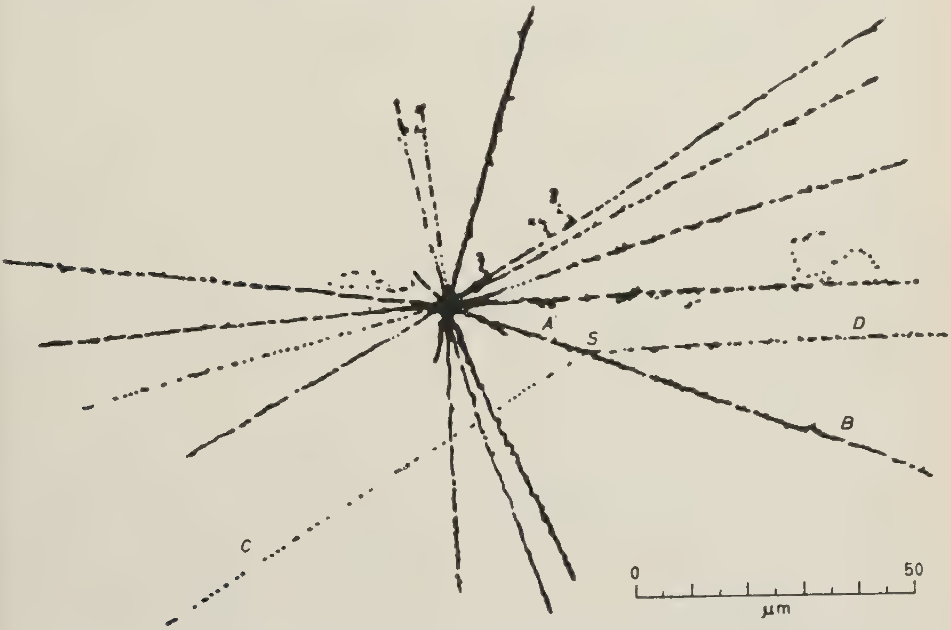

Fig. 1. – Projection drawing of a star emitting a hyperfragment (A) which decays at S into a deuteron (B), a negative π-meson (C) and a proton (D).

B, C, D, taking into account the shrinking of the emulsion, are given in Table I.

TABLE I.

$$BSC = 113.1 \pm 3^{\circ}$$
$$BSD = 23.8 \pm 3^{\circ}$$
$$CSD = 136.3 \pm 3^{\circ}$$

Thus the three prongs are almost coplanar, the track B making an angle of only $\sim 6^{\circ}$ with the plane containing C and D. The rather large errors in the measured angles are due to the steepness of the three prongs, the angles between

sisting of particle B and a weakly bound Λ^0. If D is assumed to be a proton, and C and D both secondaries of the Λ^0-particle, the Q-value for the Λ^0-decay is 35.7 ± 1.0 MeV. The event thus fits very well with a Λ^0-decay, and in the following discussion the particle D will be assumed to be a proton.

We should like to point out the very great similarity between the present event and the cloud chamber picture of a singly charged unstable fragment reported by ALEXANDER *et* al. (3). A

(2) G. BARONI, C. CASTAGNOLI, G. CORTINI, C. FRANZINETTI and A. MANFREDINI: *Bureau of Standard CERN Bulletin No. 9.*

(3) G. ALEXANDER, C. BALLARIO, R. BIZZARRI, B. BRUNELLI, A. DE MARCO, A. MICHELINI, G. MONETI, E. ZAVATTINI, A. ZICHICHI and J. ASTBURY: *Nuovo Cimento*, **2**, 365 (1955).

characteristic feature of the mesonic decay of some light hyperfragments seems to be that the bound Λ^0-particle disintegrates as if it were nearly undisturbed by the accompanying particle (cf. GATTO [4]).

Assuming the Λ^0-particle to be travelling in the direction of A and B, its velocity at the decay point S is $\beta \sim 0.16$, calculated from the general dynamics of a Λ^0-decay. The corresponding ranges for a proton, deuteron or triton are $\sim 780\ \mu$m, $\sim 1560\ \mu$m and $\sim 2340\ \mu$m respectively. The observed range $1578\ \mu$m of B thus strongly indicates that the track is due to a deuteron of kinetic energy 24.85 MeV. It thus appears that the most probable interpretation of the event is that A is the track of a ^{3}H* hyperfragment decaying in flight according to the mode:

$$^3\mathrm{H}^* \to {}^2\mathrm{H} + {}^1\mathrm{H} + \pi^- .$$

Assuming this decay scheme the momentum of the ^{3}H* particle at the disintegration point S is $P = 479.1$ MeV/c, and the calculated angle between the directions of the particles A and B is $\sim 3^\circ$, which is of the order of the experimental error due to the steepness of the two tracks. The transverse momentum imparted to the residual deuteron after the decay is ~ 16 MeV/c. Calling the mass of the meson-active triton $m(^3\mathrm{H}^*)$, where

$$m(^3\mathrm{H}^*) = m(^2\mathrm{H}) + m(\Lambda^0) - \mathrm{BE}\ (\Lambda^0)$$

the binding energy BE (Λ^0) of the Λ^0-particle in the hyperfragment can be calculated from:

$$\frac{P^2}{2m(^3\mathrm{H}^*)} + m(^3\mathrm{H}^*) = E,$$

where $E = 3026.9$ MeV is the sum of the total energies of the particles B, C and D. The result gives

$$\mathrm{BE}\ (\Lambda^0) = 1.4 \pm 1.0\ \mathrm{MeV},$$

which may be compared with the energy 6.3 MeV required to remove a neutron from a normal triton.

In this calculation we have used the mass values for elementary particles and light nuclei given by BARKAS and HAHN [5], and 36.92 MeV for the Q-value in the decay of a free Λ^0-particle (FRIEDLANDER *et al.* [6]).

We believe that the main source of error in the calculation of BE (Λ^0) is due to the difficulty in measuring the true range of the π-meson because of the great fraction of the trajectory lying between the emulsion strips. Our result, therefore, is not inconsistent with a binding energy of the Λ^0-particle less than 1 MeV.

In view of the recent discovery of the hyperfragment ^{4}H* we might consider the alternative possibility of the present event being a decay according to the mode:

$$^4\mathrm{H}^* \to {}^2\mathrm{H} + {}^1\mathrm{H} + {}^1\mathrm{n} + \pi^- .$$

It can be shown, however, that no positive binding energy can be assigned to the Λ^0-particle in that case, taking into account the observed values of energies and angles. We therefore definitely exclude this possibility.

Observations of the decays in flight of unstable particles are valuable because they yield information about the mean lifetimes. Assuming a velocity of the ^{3}H* of $\beta = 0.16$ the time of flight of the particle before decay was $\sim 10^{-12}$ s.

The primary star has no other prongs giving rise to visible decays or interactions.

* * *

The authors are indebted to Professor J. HOLTSMARK and Professor R. TANGEN for providing laboratory facilities and to the Royal Norwegian Council for Scientific and Industrial Research for financial support.

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(4) R. GATTO: *Nuovo Cimento*, **2**, 373 (1955).

(5) W. BARKAS and G. HAHN: *Emulsion Tables*, Univ. of Cal., Rad. Lab. - 2579 (1954).

(6) M. FRIEDLANDER, D. KEEFE, M. G. K. MENON and M. MERLIN: *Phil. Mag.*, **45**, 533 (1954).

Gravitational Motion and Hamilton's Principle.

B. Bertotti (*)

Dublin Institute for Advanced Studies

(ricevuto il 24 Gennaio 1956)

1. – In Newtonian mechanics a complete description of any conservative system is contained in Hamilton's principle: one function in phase space, the Lagrangian, is enough for characterizing the system. The empty space equations of general relativity are also derived from a (four-dimensional) variational principle; however when matter is present they are not to be regarded any more as « field laws », but as a mere definition of energy and momentum. Yet the same integral, in virtue of its being an invariant, is still unaffected by a particular kind of variation of the g's, i.e. any one brought about by a change of frame; and this leads to the conservation laws. Is there any *direct* bearing between the two action integrals? Under which circumstances can a one-dimensional variational principle be derived straightforwardly from the *invariance property* of the four-dimensional one? We will show that this is the case for a *test-particle*; and we will get in a very simple way a « *generalized Hamiltonian principle* », which embodies two well-known results: the geodesic law and the constancy of the rest-mass.

Any infinitesimal variation of $g_{\mu\nu}$ induces in the action

$$J = \int \mathfrak{g}^{\mu\nu} R_{\mu\nu} \, \mathrm{d}^4 x \tag{1}$$

($\mathfrak{g}^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu}$; $R_{\mu\nu}$ being the contracted curvature tensor) the variation

$$\delta J = \int (\delta \mathfrak{g}^{\mu\nu} R_{\mu\nu} + \mathfrak{g}^{\mu\nu}\, \delta R_{\mu\nu})\, \mathrm{d}^4 x \,.$$

The second term, being equal to an ordinary divergence, is « virtually » irrelevant (1); a simple computation shows that the first term can be written

$$\delta J = -\int \delta g_{\mu\nu} \sqrt{-g} (R^{\mu\nu} - \tfrac{1}{2} g^{\mu\nu} R)\, \mathrm{d}^4 x \,.$$

Because of the field equations, aside from a universal constant, this is

$$\delta J = \delta \int g_{\mu\nu} \mathfrak{T}^{\mu\nu}\, \mathrm{d}^4 x = \int \delta g_{\mu\nu} \mathfrak{T}^{\mu\nu}\, \mathrm{d}^4 x \qquad (\mathfrak{T}^{\mu\nu} = T^{\mu\nu}\sqrt{-g}).$$

(*) On leave of absence from the Istituto Nazionale di Fisica Nucleare, Sezione di Milano.

(1) See, e.g., E. Schrödinger: *Space-time Structure* (Cambridge, 1950), p. 98.

For a variation (distinguished by a star) brought about by a change of frame,

$$\delta^* J = \int \delta^* g_{\mu\nu} \mathfrak{T}^{\mu\nu} \mathrm{d}^4 x = 0 \; ; \tag{2}$$

and since, by the very definition of δ^*,

$$\int (\delta^* g_{\mu\nu} \mathfrak{T}^{\mu\nu} + g_{\mu\nu} \delta^* \mathfrak{T}^{\mu\nu}) \, \mathrm{d}^4 x = 0 \, ,$$

we have also

$$\delta^* J = \int g_{\mu\nu} \delta^* \mathfrak{T}^{\mu\nu} \mathrm{d}^4 x = 0 \, . \tag{3}$$

Let us examine more closely this strange « variational principle ».

The operator δ^* is the sum of two distinct variations: the one brought about in each component by the change in the argument (let us call it Δ); and the change produced by restoring the previous argument. In symbols:

$$\delta^* \mathfrak{T}^{\mu\nu} = \Delta \mathfrak{T}^{\mu\nu} - \mathfrak{T}^{\mu\nu}{}_{,\varrho} a^\varrho \; ([2]), \tag{4}$$

if the change of frame is

$$x^\varrho \to x'^\varrho = x^\varrho + a^\varrho \, . \tag{5}$$

$\Delta \mathfrak{T}^{\mu\nu}$ is defined by

$$\Delta \mathfrak{T}^{\mu\nu} = \mathfrak{T}'^{\mu\nu}(x'^\varrho) - \mathfrak{T}^{\mu\nu}(x^\varrho) \, . \tag{6}$$

We recall also that

$$\Delta \sqrt{-g} = -\sqrt{-g} \, a^\varrho{}_{,\varrho} \, . \tag{7}$$

Then (3) becomes, by (4) and (7):

$$\int (g_{\mu\nu} \sqrt{-g} \, \Delta T^{\mu\nu} - g_{\mu\nu} \mathfrak{T}^{\mu\nu} a^\varrho{}_{,\varrho} - g_{\mu\nu} \mathfrak{T}^{\mu\nu}{}_{,\varrho} a^\varrho) \, \mathrm{d}^4 x = 0 \, .$$

With a partial integration in the last term we get:

$$\int (g_{\mu\nu} \Delta T^{\mu\nu} + g_{\mu\nu,\varrho} T^{\mu\nu} a^\varrho) \sqrt{-g} \, \mathrm{d}^4 x \, . \tag{8}$$

([2]) The comma means ordinary derivative.

2. – A rigorous definition of a test-particle requires some preliminaries. Let

$$l: \qquad x^\mu = y^\mu(\tau) \, , \tag{9}$$

be a time-like world line, in function of a parameter τ; and V_τ a three dimensional neighbourhood of l, orthogonal to it at $y^\mu(\tau)$. Let also ϱ be an invariant function of the coordinates, having the properties of the product of three Dirac's δ-functions and defined as follows: it vanishes everywhere except on l, and for any regular function $f(x^\mu)$

$$\int_{V_\tau} f(x^\mu) \varrho \sqrt{-g} \, \mathrm{d}^3 x = f(y^\mu(\tau)) \, m(\tau) \, . \tag{10}$$

A test-particle is then described by the energy-momentum tensor

$$T^{\mu\nu} = \varrho \dot{y}^\mu \dot{y}^\nu \qquad \left(\dot{y}^\mu = \frac{\mathrm{d} y^\mu}{\mathrm{d}\tau} \right) , \tag{11}$$

in the limit when the parameter $m \to 0$ for all τ's. We apologize for using here functions of τ only, the $\dot{y}$'s, as if they were functions of all the coordinates; but since $T^{\mu\nu}$ vanishes outside l, they need not be defined elsewhere, and no ambiguity can arise.

The next step is to contemplate a δ^* produced by a particular change of frame, which may be said to consist only in an arbitrary variation of l. Namely:

$$a^\mu \doteq \delta y^\mu \tag{12}$$

are now four regular functions of the coordinates, vanishing everywhere except in the neighbourhood of l and taking on it the small, arbitrary values $\delta y_0^\mu(\tau)$ (say). The field over which J is computed will be the neighbourhood of l, for a sufficiently long interval of τ. The integration will be split in a three-dimensional integration over V_τ and in another one with respect to τ. Since the g's are regular functions of the coordinates ($m \to 0$!), the first gives be-

cause of (10):

$$L = \int_{V_\tau} g_{\mu\nu} \mathfrak{T}^{\mu\nu} \mathrm{d}^3x = m\tilde{g}_{\mu\nu}\dot{y}^\mu \dot{y}^\nu , \tag{13}$$

where

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}(y^\alpha(\tau)) . \tag{14}$$

It is easily seen then that the general law (8) means in our case that

$$J = \int L(y^\alpha, \dot{y}^\alpha, \tau)\, \mathrm{d}\tau \tag{15}$$

is *stationary* for an arbitrary variation of l.

This is so because: *a*) $\Delta T^{\mu\nu}$ is simply $(\partial T^{\mu\nu}/\partial \dot{y}^\alpha)\, \delta \dot{y}^\alpha$, since ϱ is an invariant; *b*) the $g_{\mu\nu}$ are functions of the coordinates alone, and not of $\dot{y}^\alpha$; *c*) $g_{\mu\nu,\alpha}$, because of (10) can be replaced by their values at l in the integration over V_τ. (8) then reads

$$\int \mathrm{d}\tau \left(\delta \dot{y}_0^\alpha \frac{\partial}{\partial \dot{y}^\alpha} \int_{V_\tau} g_{\mu\nu} \mathfrak{T}^{\mu\nu} \mathrm{d}^3x + \right.$$
$$\left. + \ \delta y_0^\alpha \frac{\partial \tilde{g}_{\mu\nu}}{\partial y^\alpha} \int_{V_\tau} \mathfrak{T}^{\mu\nu} \mathrm{d}^3x \right) = 0 ,$$

which is just another way of stating that J is stationary.

3. – The consequence of this one-dimensional variational principle can be seen at once by going over to another parameter σ such that

$$\frac{\mathrm{d}\tau}{\mathrm{d}\sigma} = m . \tag{16}$$

The new Lagrangian is

$$L' = Lm = \tilde{g}_{\mu\nu} \frac{\mathrm{d}y^\mu}{\mathrm{d}\sigma} \frac{\mathrm{d}y^\nu}{\mathrm{d}\sigma} ; \tag{17}$$

hence J stationary means that [3]:

a) the path is geodesic;

$$b) \quad \sqrt{L'} = \frac{\mathrm{d}s}{\mathrm{d}\sigma} = m \frac{\mathrm{d}s}{\mathrm{d}\tau} = m_0 \quad \text{(say)} \tag{18}$$

is a constant of the motion, the rest-mass.

L reduces to the Hamiltonian action (kinetic minus potential energy) in the approximation of weak fields and small velocities, when the time is taken as a parameter.

* * *

I wish to thank Prof. E. SCHRÖDINGER for useful criticisms.

(3) See T. LEVI-CIVITA: *Absolute Differential Calculus* (London, 1927), p. 330.

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On the Integral Equation for the Heisenberg Current Operator.

J. C. Polkinghorne (*)

California Institute of Technology - Pasadena, California, U.S.A.

(ricevuto il 1° Febbraio 1956)

Recently Cini and Fubini [1] have shown how to derive a nonlinear equation for the Heisenberg current density that reduces in an appropriate approximation to the equation given by Low [2]. In the case of a pseudoscalar meson field satisfying the equation

$$(1) \qquad (\Box - \mu^2)\varphi(x) = j(x)\,,$$

where $j(x)$ is the current density, they make use of the equation

$$(2) \qquad [\varphi_f(x, \sigma(y)), j(y)] = 0\,.$$

Here

$$(3) \qquad \varphi_f(x, \sigma) = \int_\sigma \left\{ f(x-y')\frac{\partial\varphi}{\partial y'_\mu} - \varphi(y')\frac{\partial f(x-y')}{\partial y'_\mu}\right\} \mathrm{d}\sigma'_\mu\,,$$

the function f satisfies

$$(4) \qquad (\Box - \mu^2)f(x) = 0\,,$$

and $\sigma(y)$ is a spacelike surface passing through the point y. Equation (2) was obtained by Cini and Fubini from arguments relating to the equivalence between quantised field amplitudes and unquantised field amplitudes given by Feynman [3].

In this note we shall show that a generalisation of equation (2) (see equation (11) below) for the case $f(x) = \Delta(x)$ is the necessary and sufficient condition that the operators $\varphi(x)$ satisfy the canonical commutation relations on any spacelike surface.

(*) Commonwealth Fund Fellow; on leave of absence from Trinity College, Cambridge, England.

(1) S. Fubini: *Nuovo Cimento*, **2**, 180; M. Cini and S. Fubini: *Nuovo Cimento*, **2**, 192, 860 (1955).

(2) F. Low: *Phys. Rev.*, **97**, 1392 (1955).

(3) R. P. Feynman: *Phys. Rev.*, **80**, 440 (1950).

Equation (1) may be integrated to give

$$\varphi(x) = \varphi_\sigma(x) + \int_{-\infty}^{\infty} \Delta_\sigma(x-y)\, j(y)\, \mathrm{d}y\,, \tag{5}$$

$\Delta_\sigma(x-x')$ is the appropriate Green's function for the spacelike surface σ that has been given by YANG and FELDMAN (4). The operator $\varphi_\sigma(x)$ satisfies the free field equation and it and its first derivatives are equal to $\varphi(x)$ and its first derivatives on σ. This implies that the necessary and sufficient condition that $\varphi(x)$ satisfies the canonical commutation relations on all spacelike surfaces is that each $\varphi_\sigma(x)$ should satisfy them on its own spacelike surface σ. This implies

$$[\varphi_\sigma(x), \varphi_\sigma(x')] = \Delta(x-x')\,, \qquad \text{all } \sigma\,, \tag{6}$$

If σ is in the infinite past equation (5) becomes

$$\varphi(x) = \varphi_{\mathrm{in}}(x) + \int_{-\infty}^{\infty} \Delta_{\mathrm{ret}}(x-y)\, j(y)\, \mathrm{d}y\,, \tag{7}$$

and we may write

$$\varphi_\sigma(x) = \varphi_{\mathrm{in}}(x) - \int_{-\infty}^{\sigma} \Delta(x-y) j(y)\, \mathrm{d}y\,. \tag{8}$$

Equation (6) may now be written

$$\begin{aligned} &\int_{-\infty}^{\sigma} [\varphi_{\mathrm{in}}(x), j(y')]\, \Delta(x'-y')\, \mathrm{d}y' + \int_{-\infty}^{\sigma} [j(y), \varphi_{\mathrm{in}}(x')]\, \Delta(x-y)\, \mathrm{d}y\, - \\ &\qquad - \int_{-\infty}^{\sigma} \int_{-\infty}^{\sigma} \Delta(x-y)\, \Delta(x'-y') [j(y), j(y')]\, \mathrm{d}y\, \mathrm{d}y' = 0\,, \qquad \text{all } \sigma. \end{aligned} \tag{9}$$

Operating on equation (9) by $\delta/\delta\sigma(y)$ and using the fact that $\varphi_\sigma(x)$ is just equal to $\varphi_\Delta(x, \sigma)$ (1) we obtain

$$[\varphi_\Delta(x, \sigma(y)), j(y)]\, \Delta(x'-y) + [j(y), \varphi_\Delta(x', \sigma(y))]\, \Delta(x-y) = 0\,, \tag{10}$$

$$\text{all } x,\ x',\ \text{and } y.$$

(4) C. N. YANG and D. FELDMAN: *Phys. Rev.*, **79**, 972 (1950).
(5) M. GELL-MANN, M. L. GOLDBERGER and W. E. THIRRING: *Phys. Rev.*, **95**, 1612 (1954).

This in turn implies that

$$(11)\qquad [\varphi_\Delta(x, \sigma(y)), j(y)] = \Phi(y)\,\Delta(x-y)\,,$$

where $\Phi(y)$ is any operator function of y.

Clearly equation (6) not only implies equation (11) but is also implied by it. Thus equation (11) is the necessary and sufficient condition that $\varphi(x)$ satisfies the canonical commutation relations on every spacelike surface. Our equation differs from that used by CINI and FUBINI in having the term on the righthand side. In the approximation leading to the Low equation this gives rise to inhomogeneous terms. These represent the effect of direct meson-meson interactions such as that given by a $\lambda\varphi^4$ term.

If we operate on equation (11) by $\delta/\delta\sigma(y')$ we obtain

$$(12)\qquad [j(y'), j(y)] = 0\,, \qquad (y-y')^2 > 0,$$

which is the causality condition [5].

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Radiations From ^{178}W.

A. Bisi, S. Terrani and L. Zappa
Istituto di Fisica Sperimentale del Politecnico - Milano

(ricevuto il 3 Febbraio 1956)

The activity of ^{178}W was observed by Wilkinson [1] in bombardment of tantalum with protons of energy greater than about 25 MeV. By means of absorption measurements the decay was found to occur through orbital electron capture. The presence of a weak γ-ray of 0.270 MeV was also noticed ($\gamma/KX=0.02$). ^{178}W ($T_{\frac{1}{2}}=21.5$ d) is in equilibrium with its daughter ^{178}Ta ($T_{\frac{1}{2}}=9.35$ m). According to the results of Wilkinson the decay of ^{178}Ta occurs through orbital electron capture (94%) and β^+ emission (6%); the decay gives rise to a weak γ transition of 1.5 MeV energy ($\gamma/KX=$ $=0.03$).

A hafnium oxide target has been irradiated in the Amsterdam cyclotron with 52 MeV α-particles. Tungsten carrier was added to the irradiated sample, and then an extraction of tungsten was made. The W fraction, on investigation with a single crystal scintillation spectrometer, was found to be active. After the disappearance of short-lived activities the γ spectrum studied with a twenty channel electronic pulse analyser, showed the presence of a peak at 56.5 ± 1.5 keV energy. The intensity of this peak was followed over 200 days. The decay curve appears to be complex and can be easily broken into two straight portions. The corresponding half-lives are:

$$(T_{\frac{1}{2}})' = 22.0 \pm 0.5\,\text{d}$$

and

$$(T_{\frac{1}{2}})'' = 145 \pm 5\,\text{d}\,.$$

The two activities were therefore attributed respectively to ^{178}W and ^{181}W; (^{181}W is known to decay by orbital electron capture, $T_{\frac{1}{2}}=140$ d) [2,3]. The ratio of the two activities after the irradiation is about: 4:1.

No γ-rays were observed in the energy regions around 0.270 MeV, 0.511 MeV (positron annihilation radiation) and 1.5 MeV. If present, these γ-rays would have an intensity relative to the KX-radiation, respectively lower than $6\cdot10^{-3}$, 10^{-2}, $1.4\cdot10^{-2}$. In estimating these upper limits the detection efficiency was taken into account. The rather high values of these upper limits are due to the weakness of the active sample.

(1) G. Wilkinson: *Phys. Rev.*, **80**, 495 (1950).

(2) G. Wilkinson: *Nature*, **160**, 864 (1947).

(3) A. Bisi, S. Terrani and L. Zappa: *Nuovo Cimento*, **1**, 651 (1955).

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Meson-Meson Interaction From a Field-Theoretical-Model.

B. BOSCO and R. STROFFOLINI

Istituto di Fisica dell'Università - Torino
Istituto Nazionale di Fisica Nucleare - Sezione di Torino

(ricevuto il 4 Febbraio 1956)

Recent experiments (1) on high energy pion-nucleon scattering have shown that π^--p total cross-section has a second pronounced maximum around 900 MeV, while this maximum does not appear in the π^+-p cross-section.

In order to explain theoretically these facts one could assume the existence of a nuclear isobaric state; in this case because of the asimmetry between π^+ and π^- scattering, this state should have isotopic spin $T=\frac{1}{2}$.

However many authors (2) have given some arguments against this hypothesis; e.g. they have shown that this isobaric state should have total angular momentum $J=\frac{11}{2}$. On the other hand the recent work of MITRA (3) based on the T-D approximation of the γ^5 meson theory does not predict any such isobaric state.

These difficulties have suggested that many angular momentum states with $T=\frac{1}{2}$ contribute to this maximum.

In this connection DYSON (4) has shown that a resonant interaction between incident meson and one meson in the nucleon cloud give rise to such a situation. This resonant state should have $T=0$, J even and correspond to a relative momentum $p=250$ MeV/c.

Alternative explanations have been proposed: for example TAKEDA (5) postulates resonant state $T=1$.

In this letter we want to discuss if such a resonance can be understood from the point of view of meson theory.

The discussion will be based on a simplified hamiltonian introduced in a previous paper (6) which has the advantage of allowing an exact solution.

This hamiltonian which leads to S-wave pion nucleon scattering in fair agreement with experiment predicts a pion-pion interaction due to overlapping of the nucleon-antinucleon clouds of the two particles. The interaction Hamiltonian used in A is:

$$(1)\qquad H'=g\sum_{s=1}^{3}\sum_{pp'k}\{a_p^+b_{p'}^+\alpha_k^s\Gamma^s(p,p',k)+\text{complex conjugate}\}+\text{renormalisation term}\,.$$

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(1) A. M. SHAPIRO, C. P. LEAVITT and F. F. CHEN: *Phys. Rev.*, **92**, 1073 (1953); R. L. COOL, L. MADANSKY and O. PICCIONI: *Phys. Rev.*, **93**, 637 (1954).

(2) See, for example, C. N. YANG: *Proceedings of the Fifth Annual Rochester Conference.*

(3) A. N. MITRA: *Phys. Rev.*, **99**, 957 (1955).

(4) F. J. DYSON: *Phys. Rev.*, **99**, 1037 (1955).

(5) G. TAKEDA: *Phys. Rev.*, **100**, 440 (1955).

(6) B. BOSCO and R. STROFFOLINI: *Nuovo Cimento*, **2**, 433 (1955). In the following this paper will be denoted by A.

We introduce the pion-pion scattering $|\Pi+\Pi\rangle$ obeying to the Schrödinger equation:

$$(H_0 + H')|\Pi + \Pi\rangle = E_0|\Pi + \Pi\rangle \,. \tag{2}$$

As in A we shall use for the state $|\Pi+\Pi\rangle$ the following expansion:

$$|\Pi + \Pi\rangle = \sum_{r,s=1}^{3} \sum_{k_1k_2} \varphi^{rs}(k_1k_2)\,|\Pi^r_{k_1}\Pi^s_{k_2}\rangle + \sum_{t=1}^{3} \sum_{kp_1p_2} \psi^t(kp_1p_2)a^+_{p_1}b^+_{p_2}|\Pi^t_k\rangle + \sum_{p_1p_2p_3p_4} \chi(p_1p_2p_3p_4)\,|a^+_{p_1}a^+_{p_2}b^+_{p_3}b^+_{p_4}\rangle \,, \tag{3}$$

$|\Pi^r_{k_1}\Pi^s_{k_2}\rangle$ denotes the Kronecker product of the «physical» meson states $|\Pi^r_{k_1}\rangle$ and $|\Pi^s_{k_2}\rangle$.

We want to point out that due to the form of the interaction Hamiltonian, the expansion (3) of $|\Pi+\Pi\rangle$ containing only the three amplitudes φ, ψ and χ is a rigorous one.

Introducing the expansion (3) in (2) and using standard techniques one can get a system of three coupled integral equations for the amplitudes φ, ψ and χ. In order to obtain explicitly the form of the pion-pion interaction one has to eliminate from this system the amplitudes ψ and χ.

This can be achieved if one neglects completely the nucleon recoils (7); in this case of course we shall get only S-wave scattering, in the state $T = 0,2$.

The result of these rather cumbersome calculations is:

$$\varphi_T(h)[2W_h - E_0]U(h) = \left(\frac{g_c^2}{4\pi}\right)^2 \int L_T(h, k)\varphi_T(k)k^2\,\mathrm{d}k \,, \tag{4}$$

φ is the two mesons amplitude in the center of mass system (8);
h is the relative momentum.
T is the total isotopic spin and:

$$\left\{\begin{aligned}
L_T(h, k) &= \frac{16}{3\pi^2}\,\frac{M^3\xi^3[\frac{1}{2}T(T+1)-1]}{[1-g_c^2A(0)]^2[4M-E_0]}\,\frac{1}{1+\dfrac{g_c^2I[\frac{1}{2}T(T+1)-1]}{2\pi^2(4M-E_0)}}\,\cdot \\
&\qquad\cdot\frac{1}{\omega_hV(h)}\,\frac{N(k)}{\omega_k(W_k-2M)}\left[1-\frac{(2W_k-E_0)H(k)}{V(k)}\right], \\
U(h) &= 1-\frac{2}{3}\,\frac{g_c^2M^3\xi^3}{\pi^2}\,\frac{H(h)}{\omega_h(W_h-2M)V(h)[1-g_c^2A(0)]}, \\
V(h) &= H(h)[W_h+2M-E_0]-\frac{g_c^2M^3\xi^3}{3\pi^2\omega_h(4M-E_0)\{1+g_c^2[A(h)-A(0)]\}}, \\
H(h) &= 1-\frac{g_c^2}{3\pi^2}\,\frac{M^3\xi^3(W_h+2M-E_0)}{\omega_h(W_h-2M)^2(4M-E_0)\{1+g_c^2[A(h)-A(0)]\}}, \\
I &= \int\frac{k^2\,\mathrm{d}k}{\omega_kV(k)\{1+g_c^2[A(k)-A(0)]\}}\,.
\end{aligned}\right. \tag{5}$$

(7) This is a reliable approximation as long as we use a cut-off theory and we are interested in the low energy region.

(8) Since we use for nucleon momentum a cut-off ξM, the corresponding cut-off for the meson momentum is $2\xi M$.

Let p be the value of h for which $2W_h - E_0$ vanishes; then the equation (4) becomes:

$$\varphi_T(h) = \delta(p-h) + \frac{1}{2}\left(\frac{g_c^2}{4\pi}\right)^2 \int \frac{L_T(h, k)}{U(h)(W_h - W_p)} \varphi_T(k) k^2 \, \mathrm{d}k \; . \tag{6}$$

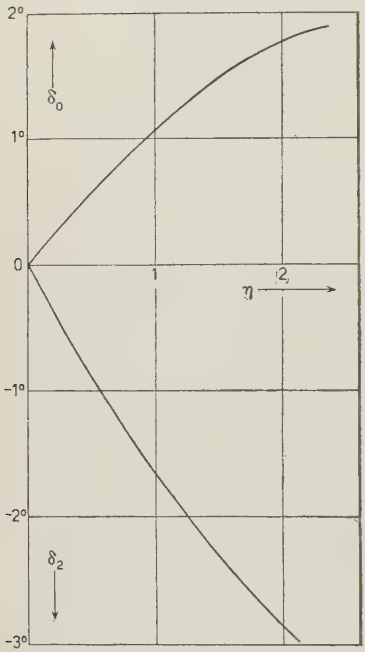

Fig. 1.

Equation (6) together with the definitions (5) shows that the pion-pion potential is attractive for $T = 0$ and repulsive for $T = 2$ [8].

There is therefore in principle the possibility of having a $T = 0$ resonance as predicted by DYSON.

A solution of (6) based on the Schmidt methods gives phase-shifts which depend very strongly on the value of the coupling constant assumed.

If one uses the parameters which fit the low energy pion-nucleon scattering S-waves namely $g_c^2/4\pi = 1$ and $\xi = 0.7$ one obtains phase-shifts of the order of some degrees and not essentially different from the Born-approximation results. These phase-shifts are given in Fig. 1. On the other hand it is sufficient to assume $g_c^2/4\pi \sim 2.5$ in order to have a $T = 0$ resonance.

The conclusion of this investigation is that although our model gives a pion-pion $T = 0$ attraction as postulated by DYSON, nevertheless the strength of this attraction is not sufficient to explain a resonance.

* * *

We are greatly indebted to Proff. M. CINI, S. FUBINI, G. WATAGHIN for helpful discussions, suggestions and kind interest throughout the course of this work. One of us (R.S.) is grateful to I.N.F.N. for financial support.

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(8) This happens for cut-off values of the order of nucleon mass and with not too large values of the coupling constant.

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A Possible Method for Determining the Spins of the Σ and of the Λ.

R. GATTO

Istituto di Fisica e Scuola di Perfezionamento in Fisica Nucleare dell'Università - Roma
Istituto Nazionale di Fisica Nucleare - Sezione di Roma

(ricevuto il 14 Febbraio 1954)

In a preceding paper (¹) a method was suggested for determining the spins of the Ξ^- and of the Λ^0 which uses the angular correlation in the cascade decay $\Xi^- \to \Lambda^0 + \pi^-$, $\Lambda^0 \to p + \pi^-$. The method requires the measurement of the angle θ between the line of flight of the Λ^0 (arising from the Ξ^- decay) and the direction of emission of its decay products in the center of mass of the Λ^0 itself. This angle can be easily inferred from the kinematical data and a sufficient statistics will directly furnish information on the spins. It is the purpose of this paper to present a method for determining the spins of the Σ^0 and of the Λ^0 which uses the angular correlation in the cascade decay $\Sigma^0 \to \Lambda^0 + \gamma$, $\Lambda^0 \to p + \pi^-$. Also for this method it is only required to measure the angle θ between the line of flight of the emitted Λ^0 and the direction of emission of its disintegration products in the Λ^0 center of mass system. A feature of these methods for determining the hyperon spins is that no knowledge at all is required of the state of polarization of the parent particle, the Ξ^- and the Σ^0 respectively. Therefore the statistics may include Ξ^-'s and respectively Σ^0's of any origin and of any energy. In fact any cascade decay of the Ξ^- and similarly any cascade decay of the Σ^0 gives a new point for the statistics. The situation is here similar to the case of the determination of the spin of the τ^+ by observing the correlations among its decay products—no knowledge at all of the polarization of the τ^+ (if it has spin $\neq 0$) is required. On the other hand, the angular correlations in reactions such as, for instance, the sequence $\pi^- + p \to Y + K$, $Y \to N + \pi$ etc., essentially depend on the polarization of the produced Y. This polarization can be specified with a certain number of parameters, which however are expected to depend on the energy and on the angle of emission of the Y. Therefore one should select hyperons produced at a given energy and at a given angle. A similar situation also occurs in the case of cascade decays for the correlation between the first and the second decay plane. In all such cases the collection of data relative to widely different experimental conditions will tend to reduce the correlation effects so that one rather obtains lower limits for the spins.

(¹) R. GATTO, *Il Nuovo Cimento*, 2, 841, 1955.

TABLE I.

Σ^0 \ Λ^0	$\frac{3}{2}+$ $(l_2=1)$	$\frac{3}{2}-$ $(l_2\mathrm{m}2)$	$\frac{5}{2}+$ $(l_2=3)$	$\frac{5}{2}-$ $(l_2=2)$	$\frac{7}{2}+$ $(l_2=3)$	$\frac{7}{2}-$ $(l_2=4)$
$\frac{1}{2}+$	1, 2 (*U*) (*m*) (*e*)	1, 2 (*F*) (*e*) (*m*)	2, 3 (*F*) (*e*)(*m*)	2, 3 (*U*) (*m*) (*e*)	3, 4 (*U*) (*m*) (*e*)	3, 4 (*F*) (*e*) (*m*)
$\frac{1}{2}-$	1, 2 (*F*) (*e*) (*m*)	1, 2 (*U*) (*m*) (*e*)	2, 3 (*U*) (*m*) (*e*)	2, 3 (*F*) (*e*) (*m*)	3, 4 (*F*) (*e*) (*m*)	3, 4 (*U*) (*m*) (*e*)
$\frac{3}{2}+$	1, 2, 3 (*U*) (*m*) (*e*) (*m*)	1, 2, 3 (*F*) (*e*) (*m*) (*e*)	1, 2, 3, 4 (*U*) (*m*) (*e*) (*m*) (*e*)	1, 2, 3, 4 (*F*) (*e*) (*m*) (*e*) (*m*)	2, 3, 4, 5 (*F*) (*e*) (*m*) (*e*) (*m*)	2, 3, 4, 5 (*U*) (*m*) (*e*) (*m*) (*e*)
$\frac{3}{2}-$	1, 2, 3 (*F*) (*e*) (*m*) (*e*)	1, 2, 3 (*U*) (*m*) (*e*) (*m*)	1, 2, 3, 4 (*F*) (*e*) (*m*) (*e*) (*m*)	1, 2, 3, 4 (*U*) (*m*) (*e*) (*m*) (*e*)	2, 3, 4, 5 (*U*) (*m*) (*e*) (*m*) (*e*)	2, 3, 4, 5 (*F*) (*e*) (*m*) (*e*) (*m*)
$\frac{5}{2}+$	1, 2, 3, 4 (*U*) (*m*)(*e*)(*m*)(*e*)	1, 2, 3, 4, (*F*) (*e*)(*m*)(*e*)(*m*)	1, 2, 3, 4, 5 (*U*) (*m*)(*e*)(*m*)(*e*)(*m*)	1, 2, 3, 4, 5 (*F*) (*e*)(*m*)(*e*)(*m*)(*e*)	1, 2, 3, 4, 5, 6 (*U*) (*m*)(*e*)(*m*)(*e*)(*m*)(*e*)	1, 2, 3, 4, 5, 6 (*F*) (*e*)(*m*)(*e*)(*m*)(*e*)(*m*)
$\frac{5}{2}-$	1, 2, 3, 4 (*F*) (*e*)(*m*)(*e*)(*m*)	1, 2, 3, 4 (*U*) (*m*)(*e*)(*m*)(*e*)	1, 2, 3, 4, 5 (*F*) (*e*)(*m*)(*e*)(*m*)(*e*)	1, 2, 3, 4, 5 (*U*) (*m*)(*e*)(*m*)(*e*)(*m*)	1, 2, 3, 4, 5, 6 (*F*) (*e*)(*m*)(*e*)(*m*)(*e*)(*m*)	1, 2, 3, 4, 5, 6 (*U*) (*m*)(*e*)(*m*)(*e*)(*m*)(*e*)
$\frac{7}{2}+$	2, 3, 4, 5 (*F*) (*e*)(*m*)(*e*)(*m*)	2, 3, 4, 5 (*U*) (*m*)(*e*)(*m*)(*e*)	1, 2, 3, 4, 5, 6 (*U*) (*m*)(*e*)(*m*)(*e*)(*m*)(*e*)	1, 2, 3, 4, 5, 6 (*F*) (*e*)(*m*)(*e*)(*m*)(*e*)(*m*)	1, 2, 3, 4, 5, 6, 7 (*U*) (*m*)(*e*)(*m*)(*e*)(*m*)(*e*)(*m*)	1, 2, 3, 4, 5, 6, 7 (*F*) (*e*)(*m*)(*e*)(*m*)(*e*)(*m*)(*e*)
$\frac{7}{2}-$	2, 3, 4, 5 (*U*) (*m*)(*e*)(*m*)(*e*)	2, 3, 4, 5, (*F*) (*e*)(*m*)(*e*)(*m*)	1, 2, 3, 4, 5, 6 (*F*) (*e*)(*m*)(*e*)(*m*)(*e*)(*m*)	1, 2, 3, 4, 5, 6 (*U*) (*m*)(*e*)(*m*)(*e*)(*m*)(*e*)	1, 2, 3, 4, 5, 6, 7 (*F*) (*e*)(*m*)(*e*)(*m*)(*e*)(*m*)(*e*)	1, 2, 3, 4, 5, 6, 7 (*U*) (*m*)(*e*)(*m*)(*e*)(*m*)(*e*)(*m*)

In the cascade $\Sigma^0 \to \Lambda^0 + \gamma$, $\Lambda^0 \to p + \pi^-$ the angular correlation for the angle θ between the line of flight of the Λ^0 and the direction of emission (in the Λ^0 center of mass system) of the π^-, $W(\cos\theta)$, can be written as

$$W(\cos\theta) = \\ = \sum_{l_1 l_1' \nu_1} T(l_1)T(l_1')(l_1 l_1' \, 1{-}1\,|\,\nu 0)(l_2 l_2 00\,|\,\nu 0) \cdot \\ \cdot\; W(ssl_1 l_1' \nu S)\, W(ssl_2 l_2 \nu \tfrac{1}{2}) P_\nu(\cos\theta)\,,$$

where S is the spin of the Σ^0 and s is the spin of the Λ^0; l_1 and l_1' may take all values permitted for the relative orbital angular momentum between the emitted Λ^0 and γ; l_2 is the relative orbital angular momentum between the emitted p and π^-; $T(l)$ are reduced matrix elements, and the other coefficients are Clebsch-Gordon and Racah coefficients written in the usual notation. The pos-

TABLE II.

S	s	The angular correlation W for the angle θ between the line of flight of the Λ^0 and the direction of emission of the π^- (in the the Λ^0 center of mass system)
$\frac{1}{2}$	$\frac{3}{2}$	$W = 1 + \xi^2 - (0.5000 - 1.7320\xi - 0.5000\xi^2)P_2(\cos\theta)$
$\frac{1}{2}$	$\frac{5}{2}$	$W = 1 + \xi^2 + (0.5714 + 0.8081\xi + 0.8571\xi^2)P_2(\cos\theta) + (-0.5714 + 2.0203\xi + 0.1429\xi^2)P_4(\cos\theta)$
$\frac{1}{2}$	$\frac{7}{2}$	$W = 1 + \xi^2 + (0.8929 + 0.4611\xi + 1.0119\xi^2)P_2(\cos\theta) + (0.1753 + 1.3581\xi + 0.5260\xi^2)P_4(\cos\theta) + (-0.5682 + 2.0538\xi - 0.0379\xi^2)P_6(\cos\theta)$
$\frac{3}{2}$	$\frac{3}{2}$	$W = 1 + \xi^2 + (0.4000 + 1.5492\xi)P_2(\cos\theta)$
$\frac{3}{2}$	$\frac{5}{2}$	$W = 1 + \xi^2 + (-0.3940 + 2.0284\xi + 0.2041\xi^2)P_2(\cos\theta) + 0.6531\xi^2 P_4(\cos\theta)$
$\frac{3}{2}$	$\frac{7}{2}$	$W = 1 + \xi^2 + (0.5102 + 1.0122\xi + 0.5952\xi^2)P_2(\cos\theta) + (-0.3673 + 1.9839\xi - 0.0195\xi^2)P_4(\cos\theta) + 0.7576\xi^2 P_4(\cos\theta)$
$\frac{5}{2}$	$\frac{3}{2}$	$W = 1 + \xi^2 + (-0.1000 - 1.1832\xi - 0.3571\xi^2)P_2(\cos\theta)$
$\frac{5}{2}$	$\frac{5}{2}$	$W = 1 + \xi^2 + (0.4571 + 1.0842\xi - 0.2041\xi^2)P_2(\cos\theta) - 0.3673\xi^2 P_4(\cos\theta)$
$\frac{5}{2}$	$\frac{7}{2}$	$W = 1 + \xi^2 + (-0.3571 + 2.0620\xi + 0.0850\xi^2)P_2(\cos\theta) + 0.6531\xi^2 P_4(\cos\theta)$
$\frac{7}{2}$	$\frac{3}{2}$	$W = 1 + \xi^2 + (0.1429 - 0.9258\xi - 0.5000\xi^2)P_2(\cos\theta)$
$\frac{7}{2}$	$\frac{5}{2}$	$W = 1 + \xi^2 + (-0.1429 - 1.4846\xi - 0.1257\xi^2)P_2(\cos\theta) + 0.1088\xi^2 P_4(\cos\theta)$
$\frac{7}{2}$	$\frac{7}{2}$	$W = 1 + \xi^2 + (0.4762 + 0.8248\xi - 0.2721\xi^2)P_2(\cos\theta) - 0.4898\xi^2 P_4(\cos\theta)$

sible multipole orders for the emitted γ-ray in the Σ^0 decay are reported in Table I for the different choices of the spins (up to $\frac{7}{2}$) and of the parities (parities are always relative to the proton). Electric transitions are denoted by (e) and magnetic transitions by (m). The favored cases are denoted by (F), the unfavored by (U).

In the favored cases one may assume that only the lowest multipole intervenes. In the unfavored cases a similar assumption would require a better justification. In the Table II we report the explicit angular correlations calculated for the cases reported in Table I, supposing that the two lowest multipoles contribute. The mixing ratio of the two multipoles is denoted by ξ, which can assume real values only. For the favored cases one may assume $\xi = 0$.

A particularly suitable situation for the measurement of this correlation would be provided by the absorption of K^- by hydrogen. In some cases the absorption may occur though $K^- + p \rightarrow \Sigma^0 + \pi^0$, and one can find kinematical criteria for distinguishing the Λ^0's arising from the Σ^0 decays from those produced directly via $K^- + p \rightarrow \Lambda^0 + \pi^0$.

LIBRI RICEVUTI E RECENSIONI

J. Dieudonné – *La Géométrie des groupes classiques*, Springer-Verlag, Berlin-Göttingen-Heidelberg, 1955, pagg. VII-115, DM. 20.

È il quinto quaderno della Nuova Serie della Collezione « Ergebnisse der Mathematik und ihrer Grenzgebiete » ed appartiene alla serie « Teoria dei Gruppi », curata da R. Baer.

Il presente volume è, in certo senso, un seguito ed un ampliamento di una parte dell'opera di B. L. van der Waerden, uscita nella stessa collezione nel 1935, e tiene conto dei nuovi lavori sull'argomento; esso tratta, tra i gruppi di trasformazioni lineari, la teoria dei gruppi classici, adottando, per quanto possibile, linguaggio e concezioni di natura geometrica. La trattazione, che presuppone le nozioni fondamentali di algebra lineare, è condotta senza dimostrazioni, fornendo tuttavia un cenno dei procedimenti dimostrativi. – I capitoli sono: I. Collineazioni e correlazioni; II. Struttura dei gruppi classici; III. Caratterizzazioni geometriche dei gruppi classici; IV. Automorfismi ed isomorfismi dei gruppi classici. Chiudono l'opera: *a*) una tabella delle notazioni, in una sezione della quale sono date comparativamente le notazioni di Dickson, di van der Waerden e dell'Autore; *b*) un indice delle definizioni e dei teoremi principali; *c*) una bibliografia, che si richiama all'opera citata di van der Waerden per quanto riguarda i lavori anteriori al 1935 ed è amplissima per il periodo successivo. – I gruppi trattati portano i nomi classici quali: gruppo lineare generale, gruppo delle collineazioni (proiettive), gruppo unitario, ecc., ma, si noti bene, la trattazione è fatta a partire da un corpo base arbitrario, senza escludere i casi dei corpi sghembi e dei corpi di caratteristica due, i quali, com'è ben noto, presentano spesso fenomeni essenzialmente nuovi rispetto alla teoria classica in senso stretto, e in ogni caso si scostano alquanto da tale teoria.

M. Benedicty

R. Courant – *Vorlesungen über Differential- und Integralrechnung* I. *Funktionen einer Veränderlichen.* Springer-Verlag, Berlin-Göttingen-Heidelberg, 1955, 3. Aufl. 450 S. mit/126 Textfiguren.

È questa la terza edizione del primo volume (Funzioni di una variabile) delle ormai classiche *Lezioni di Calcolo Integrale e Differenziale* di R. Courant, apparse per la prima volta nell'ormai lontano 1927, e tradotte in seguito in differenti lingue straniere. La presente edizione, benchè ampiamente rimaneggiata e migliorata, non presenta differenze essenziali rispetto alle precedenti, e non sarà certamente l'ultima di quest'opera, così preziosa, oltre che per il matematico puro, per tutti coloro che della matematica si servono in tutte le sue applicazioni, alla Fisica, alle Scienze Naturali, all'Ingegneria, e nella quale i concetti più astratti sono presentati nel modo più intuitivo e vicino alle applicazioni, senza

che peraltro il rigore ne scapiti. Le novità più importanti in questa 3ª edizione, che è aumentata di circa 40 pagine rispetto alla prima, si notano nel capitolo IX, sulle serie di Fourier, dove, fra l'altro, la sviluppabilità in siffatte serie per una funzione $f(x)$ è dimostrata sotto ipotesi meno restrittive (continuità a tratti della funzione e della sua derivata prima) di quelle ammesse nelle edizioni precedenti (continuità a tratti della funzione e delle sue derivate prima e seconda). Notevole in questo stesso capitolo IX l'appendice sui polinomi di Bernoulli e le loro applicazioni, pure nuova rispetto alle precedenti edizioni, e, nel capitolo VI sulla formula di Taylor, l'appendice sul teorema di Weierstrass sull'approssimazione di una funzione continua mediante polinomi.

Per comodità del lettore, diamo un rapido sommario dell'opera: Capitolo I: Premesse (Il continuo numerico, concetto di funzione, successioni, limiti per una successione e per una funzione [di una variabile], continuità). – Capitolo II: Concetti fondamentali del calcolo integrale e differenziale (Integrale definito, derivata, integrale indefinito, funzioni primitive e teoremi fondamentali del calcolo differenziale e integrale, ecc.). – Capitolo III: Derivazione e integrazione delle funzioni elementari (regole di derivazione e di integrazione, funzioni inverse, funzioni composte, massimi e minimi, logaritmo ed esponenziale, funzioni iperboliche, ordini di infinitesimo e di infinito). – Capitolo IV: Ulteriore costruzione del calcolo integrale (Regole di integrazione, integrazioni di funzioni razionali e delle trascendenti elementari, cenni sugli integrali ellittici, estensioni del concetto di integrale, integrali impropri). – Capitolo V: Applicazioni (geometriche e meccaniche). – Capitolo VI: La formula di Taylor e l'approssimazione di funzioni mediante funzioni razionali intere (logaritmo e arcotangente, formula di Taylor, applicazioni, teorema di Weierstrass, zeri, infiniti ed espressioni indeterminate, interpolazione). – Capitolo VII: « Excursus » sui metodi numerici (integrazione numerica, applicazioni del teorema del valor medio e della formula di Taylor, risoluzione numerica di equazioni, formula di Stirling). – Capitolo VIII: Serie ed altri processi infiniti (Concetto di convergenza, criteri di convergenza, convergenza uniforme e non uniforme, serie di potenze, sviluppi in serie di potenze, serie a termini complessi, prodotti infiniti). – Capitolo IX: Serie di Fourier. – Capitolo X: Le equazioni differenziali dei più semplici fenomeni oscillatori meccanici e fisici.

V. DALLA VOLTA

P. SAMUEL – *Méthodes d'Algèbre abstraite en Géométrie algèbrique.* Springer Verlag, Berlin-Göttingen-Heidelberg 1955, pag. IX-133, DM. 23.60.

È il quarto quaderno della Nuova Serie della collezione « Ergebnisse der Mathematik und ihrer Grenzgebiete » ed appartiene alla serie « Geometria algebrica », curata da B. SEGRE.

Il volume è un'esposizione completa ed oltremodo generale dei primi elementi della Geometria algebrica astratta, cioè della Geometria algebrica trattata coi metodi dell'Algebra astratta. Le caratteristiche principali del volume sono, a nostro avviso, quelle di seguire le più importanti teoria odierne e di condurre ogni argomento il più avanti possibile senza ipotesi limitative sul corpo base. Il volume, sui criteri espositivi e sulla forma tipografica del quale non abbiamo nulla da obiettare, richiede esplicitamente la conoscenza preliminare di elementi, anche un po' avanzati, di algebra astratta, mentre non presuppone conoscenze di Geometria algebrica; esso non è tuttavia dedicato al principiante per offrirgli (son parole dell'Autore) « un tableau attrayant de la Géométrie algébrique; son object

est d'être utile à l'usager, et celui-ci n'a plus besoin d'être convaincu ». Ci si permetta di aggiungere che per l'esperto questo libro è veramente un « tableau attrayant ».

I capitoli sono i seguenti: I. Théorie globale élémentaire (nel quale si danno le definizioni e le prime proprietà degli insiemi e delle varietà algebriche, dei concetti di proiezione, intersezione, prodotto, varietà virtuale, corrispondenza); II. Géométrie algébrique locale. Multiplicités d'intersection (Anello locale, punti normali, punti semplici, enti tangenti, teoria locale e globale dell'intersezione). Il volume si chiude con: *a*) un « Rappel algébrique », nel quale sono dettagliatamente richiamate le nozioni algebriche necessarie alla comprensione del testo, rinviando a lavori originali o a trattati; *b*) un « Annexe historique », breve e schematico, corredato da una stringata bibliografia; *c*) un « Annexe terminologique »; *d*) un indice alfabetico degli argomenti.

« L'Annexe terminologique » è una preziosissima paginetta nella quale, su otto colonne, si danno comparativamente i termini usati dall'Autore e dai più illustri rappresentanti della Geometria algebrica per esprimere i concetti più importanti della materia; e, allo stato attuale della teoria, un tal raffronto è più che utile. Prescindendo da veniali errori di stampa, a questo « Annexe » ci permettiamo di fare le seguenti due osservazioni: *a*) l'espressione « per un punto generico = generalmente » è correttamente definita nel senso della scuola geometrica italiana, *purchè* anche nella sua frase definitrice (pag. 34-35) l'espressione « x désignant un point d'une variété V » sia intesa nel senso della geometria italiana, cioè, con linguaggio astratto, che « x sia un punto algebrico di V »; se, come è sottinteso nel testo, x è un punto di dimensione qualsiasi si ha ancora una lieve differenza tra il « generalmente » di SEVERI e il « presque partout » di SAMUEL; *b*) la frase « projectivement normale », contrariamente a quanto afferma l'Autore, è definita nella scuola geometrica italiana, sia pure per k chiuso e di caratteristica zero, dalla frase (Pr) (pag. 27): quest'ultima è una condizione necessaria e sufficiente perchè valga la definizione algebrica: trattasi quindi dello stesso concetto.

M. BENEDICTY

H. HARTMANN – *Die Chemische Bindung*. Drei Vorlesungen für Chemiker; 105 pag. e 57 figure. Springer Verlag, Berlin-Göttingen-Heidelberg, 1955.

Il volume è una rielaborazione di alcune conferenze tenute dall'Autore nella Università di Francoforte s.M. ai giovani studenti di Chimica.

La materia è suddivisa in tre parti, delle quali si indica sommariamente il contenuto.

Parte I. Illustrazione schematica del comportamento corpuscolare ed ondulatorio dell'elettrone e relazione tra i due aspetti. La quantizzazione dei livelli energetici dell'elettrone. Le funzioni d'onda ed i livelli energetici degli atomi. Il sistema periodico degli elementi. La regola di Hund.

Parte II. La molecole-ione H_2^+. Gli stati di valenza degli atomi di C, N ed O ed i legami che da tali stati prendono origine.

Parte III. I legami nei cristalli ionici. Le molecole biatomiche e poliatomiche. L'elettronegatività. L'energia di risonanza e la coniugazione. Le catene polieniche. Il legame metallico.

L'Autore vuole aiutare i giovani studenti di chimica a conoscere la parte fondamentale della teoria della loro scienza attraverso una introduzione elementare alla teoria del legame chimico, affiancando una pubblicazione in lingua tedesca, finora mancante, ad altre dello

stesso tipo. È sua dichiarata intenzione tuttavia di differenziarsi ponendo l'accento sull'aspetto ondulatorio più che su quello corpuscolare, ed utilizzando concetti della meccanica classica per illustrare il legame chimico fin dove è possibile. Ciò viene realizzato servendosi soprattutto delle analogie formali che offre la teoria delle corde vibranti e dei sistemi oscillanti. Naturalmente, trattandosi di conferenze che devono abbracciare nel breve spazio di qualche ora una materia vastissima, il legame tra le varie parti è affidato all'intuizione ed alla immaginazione dell'uditore, sollecitate dall'abilità dell'oratore.

Sia lecito tuttavia dichiararci piuttosto scettici sulla utilità di tali tentativi. Il problema di familiarizzare i giovani chimici con la moderna teoria della chimica può risolversi offrendo loro, nei corsi universitari, gli elementi fondamentali della meccanica quantistica in maniera razionale e soddisfacente dal punto di vista fisico-matematico. Le altre soluzioni, come quella presentate dall'Autore, sembra possano ingenerare confusione ed illudere i giovani di possedere nuove cognizioni; quindi mascherare la crisi invece di contribuire a risolverla.

LEONELLO PAOLONI

Direttore responsabile: G. POLVANI

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